RETENTION RATIO REGULATION OF BANK ASSET SECURITIZATION

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To evaluate the welfare effect of the recently proposed financial regulatory measures of bank asset securitization-retention ratio regulation and information disclosure regulation, we build up a two-period model with asymmetric information on asset quality between a risk averse originating bank and a continuum of risk averse investors. We find that it is impossible for a flat rate retention ratio requirement to be optimal for all markets. Although both regulatory proposals are effective in reducing investors’ informational loss, neither can improve upon the unregulated case unconditionally, due to the associated regulatory cost: retention ratio regulation aggravates adverse selection problem due to “information destruction effect”, and information disclosure requirement leads to a distortion in securitization intensity to send positive signals. However, information disclosure regulation complements retention ratio regulation when investors are relatively risk averse.

Keywords: Retention Ratio Regulation, Information Disclosure Regulation, Asset Securitization, Adverse Selection.

1. INTRODUCTION

Ever since the 2007-08 financial crisis, large volume of bank asset securitization1 by commercial banks and other financial institutions has been criticized by economists and regulatory authorities as a major contagion factor in the crisis, and measures have been proposed to regulate securitization of banks.

While the removal of credit risk from banks’ balance sheets is not a new phenomenon2, the recent wave of securitization differs from previous ones for the widespread use of sophisticated techniques, facilitating banks to transfer credit risk of more opaque assets. As documented by Duffie (2008), information associated with structured securities are much more opaque than traditional securities: even specialists in CDOs are currently ill equipped to measure the risks and fair valuation of tranches that are sensitive to default.

Because banks are in a better position to judge the true quality of their loan portfolio, outside investors will require a lemon discount on the price of the assets that are sold. Investors will therefore offer an average market price that is on average below the true valuation of the reference portfolio under symmetric information. Consequently, high quality assets tend to be driven out of the market by riskier assets and the uninformed investors will suffer an unexpected informational loss, or more severely the market for loan securitization completely breaks down; this is the standard adverse selection story in the lemons market (Alkerlof, 1970).

To regulate banks’ securitization activities and protect the uninformed investors from suffering the informational loss in the opaque market environment, several measures have been proposed by financial regulatory authorities in United States and the European Union, two most prominent of which are credit retention ratio requirement (R Regulation) and information disclosure requirement (D Regulation). In the Credit Risk Retention Act (as part of the Dodd-Frank Wall Street Reform and Consumer Protection Act of 2009, signed into law on July 21, 2010), US regulatory authorities impose a 5% retention ratio requirement.

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1 According to the Security Industry and Financial Markets Association (SIFMA), between 1997 and 2008 the outstanding value of ABSs in United States increased by more than five times, from US$ 404 billion to 2,671 billion. Although it reduced to 2,429 billion in 2009, it remains a significant volume.
2 A securitization wave had already taken place in US in the 1980s, when the ratio of loan sales to total assets rose from 1.6% to 9.0% (Berger and Udell, 1993).
requirement\(^3\) on (1) creditors that originate and transfer loans to third parties and (2) securitizers of ABS. Similar to the US proposal, the European Union has also adopted the proposal requiring originators to hold at least 5% of the securitized portfolio\(^4\).

Besides the retention ratio requirement, regulatory authorities also require originating creditors or securitizers to disclose information concerning the transaction so as to mitigate informational asymmetry. US financial regulatory reform mandates the Securities and Exchange Commission (SEC) to issue rules for originating creditors and securitizers to disclose of information regarding the assets backing each tranche or class of ABS (including loan-level details).

However, it is uncertain whether the proposals of R Regulation and D Regulation are based on solid theoretical ground, and thus we aim to provide a theoretical framework to evaluate the welfare effect of these regulatory proposals comparing with the unregulated case. We claim that securitization itself is not the evil that should be disposed off; the opaqueness associated with securitization is. The objective of regulation should not be to simply decrease securitization intensity, but rather to correct the distorted structure of securitization resulted from asymmetric information.

In a two-period model with a risk averse originating bank and dispersed risk averse investors, we find that if unregulated, asset securitization will lead to both a \textit{level distortion} and a \textit{structural distortion}: lemons premium demanded by the uninformed investors reduces liquidity of assets, and the bank tends to over-sell assets of relatively low quality. This finding conforms with the evidence of Drucker and Puri (2009), who found that the probability of loan selling is higher for riskier borrowers (junk rated, higher leverage, lower distance-to-default, and lower net income-to-asset ratio). Using Italian bank loan data, Affinito and Tagliaferri (2010) empirically investigated the determinants of securitization, and found that banks that burdened with troubled loans are more likely to perform securitization.

It is found that a flat rate of retention ratio can optimize expected social welfare only with probability 0; optimal retention ratio should vary with relative risk aversion of the bank and investors. More importantly, both R Regulation and D Regulation are effective to correct the \textit{structural distortion} and thus reduces investors’ informational loss. However, neither regulatory measure comes costlessly; new distortions are incurred to correct the original distortion. R Regulation only scratches the surface of the \textit{structural distortion}, because it worsens the adverse selection problem due to “information destruction effect”\(^5\), and thus aggravates the \textit{level distortion}. D regulation entails a signalling cost for the bank: it has to distort the securitization intensity away from the optimal level to signal asset quality. Regulation can improve social welfare only when the bank is relatively more risk averse, and D Regulation complements R Regulation to improve social welfare if market investors are relatively more risk averse.

Retention ratio regulation has not been widely discussed in the literature. Fender and Mitchell (2009) prove that if the retained equity tranche is thick enough, retaining the equity tranche is likely to best align incentives between the originator and outside investors. However, in Fender and Mitchell (2009), cost of asset securitization the is moral hazard problem that banks that have transferred much of its exposure will have less incentive to monitor the loans and thus the risk taking behavior of the borrowers. The focus of our paper is the distortions by adverse selection problem. We find that D Regulation is

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\(^3\)On November 10, 2009, Senator Chris Dodd released the Restoring American Financial Stability Act of 2009, containing an alternative proposal for regulating ABS securitization to require a 10% retention ratio.

\(^4\)The retention mechanism is not explicitly stipulated in this act; it states that “the federal banking agencies should have authority to specify the permissible forms of required risk retention”. European Parliament specifies that the form of retention may be chosen from three options: “5% of the nominal value of each of the tranches sold or transferred to the investors” (i.e. vertical slice); “retention of randomly selected exposures, equivalent to no less than 5% of the nominal amount of the securitized exposures”; “retention of the first loss tranche and, if necessary, other tranches having the same or more severe risk profile and not maturing any earlier than those transferred or sold to investors, so that the retention equals in total not less than 5% of the nominal value of the securitized exposures.”
better than R Regulation if investors are relatively conservative. Since investors of junior tranches (BBB to AA), such as insurance companies, asset managers, and hedge funds, are generally more risk seeking comparing with buyers of senior tranches (AAA), such as SIVs, ABCP conduits managers and pension funds, our finding implies that the first-loss piece retention mechanism is basically proclaimed, which is similar to Fender and Mitchell (2009).

Our analysis of D Regulation is in a much similar way to Leland and Pyle (1977) and DeMarzo (2005). Leland and Pyle (1977) use a signaling model to show how agency cost of informational asymmetry can be mitigated in the context of a partial firm sale. In Leland and Pyle (1977), only sellers are risk averse; but both the seller (the bank) and the buyers (investors) are risk averse in our model. This generates richer findings, since risk aversion of investors makes the signalling cost important in our model and thus contrary to Leland and Pyle (1977) signalling may not necessarily improve things. DeMarzo (2005) claims that if assets are not only pooled but also tranched into different risk categories, banks can signal the quality of the sold loan portfolio by retaining interests in the equity tranche. The bank in DeMarzo (2005) is risk neutral and thus the benefit of securitization does not arise from risk diversification as in our model, but from the liquidity preference of the bank.

The remainder of this essay is arranged as follows. Section 2 describes basic model setup. Section 3 presents the benchmark scenario in absence of asymmetric information. Then the unregulated equilibrium with asymmetric information and the associated (level and structural) distortions are examined in Section 4. Sections 5 evaluates welfare effects of retention ratio regulation and information disclosure regulation, and explore conditions under which information disclosure regulation can complement retention ratio regulation. Section 6 concludes.

2. MODEL SETUP

There is a risk averse bank\(^5\) with a mean-variance utility function \(EU(W) = E(W) - \frac{1}{2} \gamma Var(W)\), where \(\gamma\) is the absolute coefficient of risk aversion, and \(W\) is the random wealth at the end of last period.

**Figure 1. Timing**

At \(t = 0\), the bank has a balance sheet with assets of value \((D + K)\), and liabilities of \(D\) deposits and \(K\) capital. Suppose there is no capital adequacy requirement, or equivalently, capital requirement is non-binding for the bank\(^6\).

The asset side of the bank’s balance sheet consists of a riskless asset valuing \((D + K - X)\) (with a constant rate of return \(r_f\)) and a continuum of risky assets indexed by \(j \in [0, 1]\), each of amount \(X\) so that risky assets have a total amount of \(\int_0^1 Xdj = X\). All assets have the same maturity with the deposit, originated at \(t = 0\) and mature at \(t = 2\). Deposit repayment rate is \(r_D\); retained earnings (cash) and capital have a gross rate of interest of 1.

\(^{5}\)Rochet (2008) notices that it is difficult to rationalize assuming risk aversion of banks. Risk aversion can be rationalized as in Froot et al (1993) and Froot and Stein (1998): the reason for value maximization banks to be concerned about not only the expectation but the entire distribution of wealth, is that a wealth constrained bank faced with new investment opportunities of concave return but incurs convex cost of external funding, will behave like a risk averter and generate a rationale for risk management. Loosely speaking, risk aversion of banks with respect to its internal wealth arises from costly external funding.

\(^{6}\)As mentioned in the introduction, benefit of securitization in our model comes from risk diversification as Froot et al (1993), rather than from balance sheet management to ease capital relief.
A representative risky asset with a gross rate of return of \( \tilde{r} \tilde{r} \sim N(\tilde{r}, \sigma^2) \) at \( t = 2 \), is characterized by quality \( \tilde{r} \) at \( t = 1 \), known only to the originating bank; investors believe that \( \tilde{r} \) is randomly drawn from a uniform distribution \( [\underline{r}, \bar{r}] \), and \( \underline{r} > r_f > 1 \). At \( t = 1 \), \( q \) proportion of the risky asset is securitized at price \( p \), and sold to a continuum of identical risk averse investors, indexed by \( i \in [0, 1] \), each of whom has a mean-variance utility function with absolute risk aversion coefficient \( \lambda \). The face value of each unit of asset is normalized to be \( 1 \), and \( p \) is the market valuation for each unit of asset.

At \( t = 2 \), the return \( \tilde{r} \) of risky assets realize, and all uncertainties are dissolved and payments are made. Suppose the regulator can neither observe the asset quality, nor can it force the bank to truthfully disclose asset quality. Otherwise, a benevolent regulatory authority can simply disclose or require the originators to disclose information concerning asset quality to the market, which obviously can restore efficiency. However, it may be forbiddingly costly to do this with sophisticated securitization procedures, and thus we assume that the regulators can only impose a retention ratio requirement or require the originators to disclose their retention ratio.

In what follows, we analyze the regulatory rule of a lower bound of retention ratio requirement by analyzing the rule that stipulates an upper bound of securitization intensity \( q \), since if \( q \) can be read by the market investors as noiselessly reflect quality of the underlying asset, disclosed retention ratio is redundant in the sense that the information contained in retention ratio requirement is useless in increase the precision of investors’ valuation of securitized assets since \( q \) is a sufficient statistics for \( \tilde{r} \).

### 3. The Benchmark: Symmetric Information Case

The benchmark in our model is the decentralized equilibrium with symmetric information. The reason why the centralized equilibrium is not taken as benchmark, is that the objective of both regulatory measures to be analyzed in later sections is to mitigate efficiency distortions associated with asymmetric information on asset quality, rather than to correct the efficiency loss resulted from other market frictions making agents risk averse. Therefore, we cannot expect the regulatory proposal aiming to one sort of market friction to improve upon efficiency due to other market frictions.

#### 3.1. The Equilibrium with Symmetric Information

For convenience of exposition, define the market equilibrium where securitization intensity \( q \) can be taken as a signal of asset quality, and then consider other scenarios as its special case. Denote the information set of investors derived from

\[
q \Omega, \text{ and } \Omega = \{q\} \text{ if } q \text{ acts as a signal.}
\]

Consider investor \( i \)'s optimal demand for securitized asset \( y_i \),

\[
\max_{y_i} \{E(1 \cdot y_i \cdot \tilde{r} - py_i | \Omega) - \frac{1}{2} \lambda \cdot Var(1 \cdot y_i \cdot \tilde{r} - py_i | \Omega) \}
\]

\[
= E_r \{E[y_i(\tilde{r} - p)| \Omega, \tilde{r}]\} - \frac{1}{2} \lambda \cdot \{Var_r[E(y_i(\tilde{r} - p)| \Omega, \tilde{r})] + E_r[Var(y_i(\tilde{r} - p)| \Omega, \tilde{r})]\}
\]

(3.1)

\[
y_i \cdot [E(\tilde{r}| \Omega) - p] - \frac{1}{2} \lambda y_i^2 \cdot [Var(\tilde{r}| \Omega) + \sigma^2]
\]

where law of iterated expectation and law of total variance have been utilized in (3.1), and it is straightforward to notice that the presence of asymmetric information qualifies as an element of uncertainty to investors, who thus require a premium for that.

Since the end of period wealth \( \tilde{W} \) of the bank is \( \tilde{W} = (D + K - X)r_f + q \cdot X \cdot p(\tilde{r}) + (1 - q)X(\tilde{r} - r_f) - D \cdot r_D \), the bank’s problem is to determine optimal securitization intensity to maximize it expected utility derived from \( \tilde{W} \),

(3.2)

\[
\max_q Kr_f + D(r_f - r_D) + qX \cdot [p(\tilde{r}) - r_f] + (1 - q)X(\tilde{r} - r_f) - \frac{1}{2} \gamma \sigma^2 X^2 \cdot (1 - q)^2
\]

Therefore, the competitive market equilibrium is as defined in Definition 3.1.
Definition 3.1 Given \( D, X; r_D, r_f; \lambda, \gamma; \sigma^2 \), a competitive market equilibrium is the price and securitization intensity \( \{ p, q \} \) and security demand \( \{ y_i \}_{i \in [0,1]} \) so that:

1. Given the market price \( p \), securitization intensity \( q \) maximizes the bank’s expected utility (3.2);
2. Given the information set \( \Omega \) derived from securitization intensity \( q \), security demand \( y_i \) of each investor \( i \in [0,1] \) maximizes its expected utility (3.1);
3. Security market clears: \( \int_0^1 y_i di = qX \).

In absence of private information, the benchmark competitive market equilibrium is summarized in Proposition 3.1. All proofs are relegated to the Appendix.

Proposition 3.1 When asset quality is observable to both the originating bank and investors, the bank would like to securitize a constant proportion of its assets for any \( \hat{r} \in [\hat{r}, \bar{r}] \):

\[
(3.3) \quad p_B = \hat{r} - \lambda \sigma^2 X \cdot \frac{\gamma}{\gamma + 2\lambda} \quad q_B = \frac{\gamma}{\gamma + 2\lambda}
\]

Note that even if \( p_B < 0 \), it is still optimal for the bank to securitize the asset than not. Since assets are fairly priced in the market, this is basically a risk sharing problem; the bank and investors as a whole assume a constant proportion of the riskiness associated with uncertainty rate of returns of any asset quality, i.e. \( q_B \) is independent of \( \hat{r} \).

Moreover, it is straightforward to notice that \( \frac{\partial q_B}{\partial \gamma} > 0 \), \( \frac{\partial q_B}{\partial \lambda} < 0 \). Intuitively, if the more risk averse the sector is, the less risky assets it is supposed to hold.

It is important to note that the regulator can neither observe the asset quality, nor can it force the bank to truthfully disclose asset quality. Therefore, the regulator aims to maximize expected social welfare on all asset quality uniformly distributed over \( [\hat{r}, \bar{r}] \).

\[
(3.4) \quad EW_B = X \cdot \frac{\hat{r} + \bar{r}}{2} + [(K - X) r_f + D(r_f - r_D)] - \frac{1}{2} \gamma \sigma^2 X^2 + \frac{1}{2} \sigma^2 X^2 \cdot \frac{\gamma^2 (\gamma + 3\lambda)}{(\gamma + 2\lambda)^2}
\]

4. UNREGULATED EQUILIBRIUM IN PRESENCE OF ASYMMETRIC INFORMATION

4.1. The Unregulated Market Equilibrium

In the unregulated scenario with asymmetric information, securitization intensity \( q \) is unobservable to the market. The unconditional variance of asset quality \( \hat{r} \) is denoted as \( \eta^2 \equiv Var(\hat{r}) = \frac{(\bar{r} - \hat{r})^2}{12} \).

In what follows, we make the following assumption to rule out the case with less severe adverse selection problem, which can be analyzed in quite a similar way.

Assumption A: \( \gamma < \frac{\hat{r} - \bar{r}}{2\sigma^2 X} \).

Although the optimal \( q \) decided by the bank is unobservable to the market, investors still have information available to update their valuation: the information whether \( q = 0 \) or not can be utilized. Thus information set of investors is \( \Omega = \{ q > 0 \} \). Updating valuation based on this information makes \( q \) sub-optimal for the bank, who adjusts the optimal securitization accordingly. By an infinite rounds of interactions between market conjecture of the asset quality and response of the originator, equilibrium price and quantity are summarized as in Proposition 4.1.

Proposition 4.1 Under Assumptions A, when only the originating bank knows the true asset quality and there is no regulatory requirement on \( q \), the unregulated competitive market equilibrium is:

\[
(4.1) \quad p_U = \begin{cases} 
\hat{r} + \gamma \sigma^2 X - \lambda q_U X \cdot (\eta^2_U + \sigma^2), & \text{if } \underline{r} < \hat{r} < \hat{r}_U \\
0, & \text{if } \hat{r}_U < \hat{r} < \underline{r}
\end{cases}
\]

and
If \( \gamma < 2\lambda(1 + \frac{1}{3}\gamma^2\sigma^2X^2) \),

\[
q_U = \begin{cases} 
\frac{\hat{r} + 2\gamma\sigma^2X - \hat{r}}{2\lambda\gamma\sigma^2X + \gamma\sigma^2X}, & \text{if } \underline{r} < \hat{r} < \hat{r}_U \\
0, & \text{if } \hat{r}_U < \hat{r} < \bar{r} 
\end{cases}
\]

If \( \gamma > 2\lambda(1 + \frac{1}{3}\gamma^2\sigma^2X^2) \),

\[
q_U = \begin{cases} 
1, & \text{if } \underline{r} < \hat{r} < \underline{r} + \gamma\sigma^2X - 2\lambda\gamma\sigma^2X^2, \eta_U^2 + \sigma^2 \\
\frac{\hat{r} + 2\gamma\sigma^2X - \hat{r}}{2\lambda\gamma\sigma^2X + \gamma\sigma^2X}, & \text{if } \underline{r} + \gamma\sigma^2X - 2\lambda\gamma\sigma^2X^2, \eta_U^2 + \sigma^2 < \hat{r} < \hat{r}_U \\
0, & \text{if } \hat{r}_U < \hat{r} < \bar{r} 
\end{cases}
\]

where \( \eta_U^2 \equiv \frac{1}{3}\gamma^2\sigma^4X^2 \), \( \hat{r}_U \equiv \underline{r} + 2\gamma\sigma^2X \).

Moreover, it can be also ascertained that the bank prefers to securitize \( q \) proportion of its asset at price \( p \) than not to securitize at all if and only if \( \underline{r} < \hat{r} < \hat{r}_U \). Marketability of bank assets is dependent on investors’ willingness to invest, which is in turn closed related to their risk attitude \( \lambda \). If \( \lambda \) is relatively small, investors are ready to assume more risky assets and thus the bank can sell off its worst assets, i.e. \( q_U = 1 \) for \( \underline{r} < \hat{r} < \underline{r} + \gamma\sigma^2X - 2\lambda\gamma\sigma^2X^2, \eta_U^2 + \sigma^2 \) if \( \lambda < \frac{\gamma^2(1 + \frac{1}{3}\gamma^2\sigma^2X^2)}{2(1 + \frac{1}{3}\gamma^2\sigma^2X^2)} \).

Intuitively, the market for securitized assets partially breaks down due to the well-known lemons effect. Different from the traditional lemons market with risk neutral informed sellers, where the market completely break down, i.e. all assets except that of the worst quality are driven out of the market, in our model, assets with quality \( \underline{r} < \hat{r} < \underline{r} + 2\gamma\sigma^2X \) remain in the market. Keeping assets with quality \( \underline{r} < \hat{r} < \underline{r} + 2\gamma\sigma^2X \) is sub-optimal for the risk averse bank, because of the undissolved uncertainty undesired by the bank. This is captured by the market investors into their demand decision for securitized assets.

### 4.2. Distortions with Informational Asymmetry

A better knowledge of the asset quality makes the bank in a better position to utilize this private information for informational rent, which distorts social welfare in two ways: securitization level distortion, and securitization structural distortion, which arises from distortions on market valuation as well as on securitization intensity by asset opaqueness(Figures 2-1 and 2-2).

![Figure 2-1. Comparing Securitization Intensity](image1)

![Figure 2-2. Comparing Market Valuation](image2)
4.2.1. Securitization Level Distortion

Referring to Figure 2-1, the first distortion with informational asymmetry is **securitization level distortion**, i.e. relatively good assets are driven out of the market, whereas all assets should be securitized if they can be fairly priced. Level distortion occurs because an appropriate degree of securitization can benefit the social welfare since it is a way of risk sharing: it enables the bank to better diversify the credit risk and also provide purchasers with diversified investment opportunities. Put it differently, securitization itself is not the evil to be prohibited by the government; the opaqueness associated with the sophistication is what should be concentrated on. Securitization level distortion definitely decreases social welfare, and thus correcting it will increase social welfare unambiguously.

To quantitatively measure level distortion, define level distortion $LD$ as degree of adverse selection, i.e. assets that are ruled out of the securitization market.

$$LD \equiv AS_U = \bar{r} - \hat{r}_U$$

**Lemma 4.1** Level distortion of the unregulated equilibrium is more severe if the bank is less risk averse.

Intuitively, when $\gamma$ increases, the bank would like to securitize even better assets since it is more inclined to get rid of uncertainties with $\tilde{r}$. This is correctly conjectured by the market investors and captured in pricing the assets. Thus average asset quality conjectured by the market is higher, lessening the adverse selection problem.

4.2.2. Securitization Structural Distortion

To understand the second sort of distortion asymmetric information imposes on market equilibrium, it is useful to clarify two concepts of welfare measure: ex-ante and ex-post social welfare, i.e. the social welfare evaluated before and after asymmetric information is dissolved. Surely, the bank’s expected utility is unaffected no matter ex-ante or ex-post criterion is used; it matters only for investors.

The ex-ante utility of investors is:

$$U_{Investor}^{Ex-ante} = qX[E(\hat{r}|\Omega) - p(q)] - \frac{1}{2} \lambda q^2 X^2 \cdot [\text{Var}(\hat{r}|\Omega) + \sigma^2] = \frac{1}{2} \lambda q^2 X^2 \cdot [\text{Var}(\hat{r}|\Omega) + \sigma^2]$$

However, if the asset quality $\hat{r}$ is known, the ex-post utility of investors is:

$$U_{Investor}^{Ex-post} = qX[\hat{r} - p(q)] - \frac{1}{2} \lambda q^2 X^2 \cdot [\text{Var}(\hat{r}|\Omega) + \sigma^2] = \frac{1}{2} \lambda q^2 X^2 \cdot [\text{Var}(\hat{r}|\Omega) + \sigma^2] + qX[\hat{r} - E(\hat{r}|\Omega)]$$

The expected ex-post utility of investors $EU_{Investor}^{Ex-post} = EU_{Investor}^{Ex-ante} + \frac{1}{\tilde{r} - \hat{r}_U} \int_{\hat{r}_U}^{\tilde{r}} qX[\hat{r} - E(\hat{r}|\Omega)]d\hat{r}$ and it is straightforward to notice that structural distortion $SD$ of asymmetric information lies in the term

$$SD \equiv \int_{\hat{r}_U}^{\tilde{r}} qX[\hat{r} - E(\hat{r}|\Omega)]d\hat{r}$$

where $E(\hat{r}|\Omega) = \frac{\tilde{r} + \hat{r}_U}{2}$. From the definition of $SD$, it also refers to unexpected informational loss suffered by investors, or the information rent gained by the bank with its informational advantage.

In the benchmark, investors’ informational loss is clearly zero since $E(\hat{r}|\Omega) = \tilde{r}$. However, intuitively this term negative in the unregulated equilibrium: take $q_U$ as the weight of $\hat{r} - E(\hat{r}|\Omega)$, which is a decreasing function of $\hat{r}$. Therefore, assets of relative bad quality with $\hat{r} < E(\hat{r}|\Omega)$ is weighted more, but those with $\hat{r} > E(\hat{r}|\Omega)$ is weighted less (referring to Figure 2-2).

**Proposition 4.2** In the unregulated equilibrium, investors suffer informational loss so that the equilibrium involves structural distortion, which is increasing in $\gamma$ and decreasing in $\lambda$. 
This distortion is termed as “structural” because it characterizes the fact that in the unregulated market with private information, securitization structure is distorted towards relatively bad assets, which are more heavily (even fully) securitized. In the market with sufficiently severe adverse selection problem, investors’ demand based on its expectation over the coarse information set, is blown up for the worst assets; they are “cheated” by the originating bank to over-purchase these assets; presence of asymmetric information distorts securitization structure in the sense that high (low) quality assets are under(over)-securitized.

Securitization structural distortion leads to structural distortion of social welfare: expected utility is transferred from (uninformed) investors to the (informed) bank. Correcting this distortion does not necessarily increase social welfare.

It should be mentioned that the property that structural distortion increases in $\gamma$ and decreases in $\lambda$ is closely related to prior uniform distribution of asset quality. With uniform distribution, when adverse selection problem is worsened, the precision of market investors’ conjecture increases and thus “unexpected” informational loss decreases. However, it becomes clear shortly that this tend to proclaim R Regulation, and our evaluation of R Regulation remains unchanged qualitatively.

Thus the expected ex-post social welfare is the expectation of $W_{Ex}^{\text{post}}$ over $[\hat{r}, \bar{r}]$, defined as follows:

$$W_{Ex}^{\text{post}} = W_{Ex}^{\text{ante}} + qX[\hat{r} - E(\hat{r}|\Omega)]$$

$$= X\hat{r} + [(K - X)r_f + D(r_f - r_D)] - \frac{1}{2}\gamma\sigma^2X^2 + \frac{1}{2}X^2 \cdot \left\{2\gamma\sigma^2q - q^2 \cdot \left[\gamma\sigma^2 + \lambda(\text{Var}(\hat{r}|\Omega) + \sigma^2)\right]\right\}$$  

The presence of these two distortions in the unregulated equilibrium creates room for regulators to step in, and regulators should care more about the expected ex-post social welfare; focusing on expected ex-ante social welfare will unpleasantly underestimate the room for regulation.

5. REGULATED EQUILIBRIUM IN PRESENCE OF ASYMMETRIC INFORMATION

In the financial regulatory reform of United States and the European Union in 2009, retention ratio regulation (R Regulation) and information disclosure regulation (D Regulation) were proposed to be imposed on bank securitization activities. R Regulation stipulates that the originators involve at least $\alpha$ proportion of economic interest in the securitized portfolio, or equivalently, it prohibits banks from securitizing more than $\bar{q}$ proportion of its entire assets. D Regulation requires the originators disclose information relevant to the securitization transaction, or equivalently, to disclose $q$ to the market. If the optimal choice of $q$ is lower than $\bar{q}$, R Regulation is non-binding and the equilibrium is the same as in D Regulation. Therefore, we classify R Regulation as the stipulation of a flat ratio of $\bar{q}$.

Observing structural distortion characterized by $SD = \int_{\hat{r}}^{\bar{r}} qX[\hat{r} - E(\hat{r}|\Omega)]d\hat{r}$, it is straightforward to notice that both R Regulation and D Regulation are expected to effectively correct structural distortion, although functions with different mechanisms. R Regulation impose a constant weight $q = \bar{q}$ on the non-zero term $[\hat{r} - E(\hat{r}|\Omega)]$, while D Regulation makes $E(\hat{r}|\Omega) = \hat{r}$ for all $\hat{r}$. If the regulatory authority aims merely to correct structural distortion\(^7\), the regulatory proposals are indeed expected to function its role. However, it is unclear whether they are also effective to correct level distortion. This is the subject of the next two subsections.

\(^7\) There may be miscellaneous reasons for the government to care more about structural distortion, especially during financial crisis. Structural distortion implies the bank transfers bad assets out of the banking sector AND leads to a welfare loss to the purchasers, who are usually insurance companies, pension funds, hedge funds, and other institutional investors. Thus it exhibits a systemic property, which the regulators strive to avoid particularly in the period of financial crisis.
5.1. Market Equilibrium of R Regulation and D Regulation

With R Regulation, the originating bank is required to retain a certain proportion of economic interest in the securitized assets. Or equivalently, securitization intensity is no higher than an upper bound: \( q \leq \hat{q} \). As mentioned above, this regulation is separately examined by R Regulation \( q = \hat{q} \) and D Regulation, and the welfare effect of \( q \leq \hat{q} \) is in between. Therefore, with R Regulation and D Regulation, information available to the market investors are \( \Omega = \{ \hat{q} > 0 \} \) and \( \Omega = \{ q \} \), respectively.

Propositions 5.1 and 5.3 summarize the equilibrium price and securitization intensity in the market with R Regulation and D Regulation, respectively.

**Proposition 5.1** Under Assumption A, the equilibrium asset price and securitization intensity in the market with retention ratio requirement regulation are:

\[
\begin{align*}
\hat{p}_R &= \begin{cases} 
\hat{r} + \frac{1}{2} \gamma \sigma^2 X (2 - \hat{q}) - 2 \lambda \hat{q} X \sigma^2 (\sigma^2 + \eta^2_R), & \text{if} \ \hat{r} < \hat{r}_R \ \text{and} \ 0 < \hat{q} < \hat{q}_R \\
0, & \text{if otherwise}
\end{cases} \\
\hat{q}_R &= \begin{cases} 
\hat{q}_R, & \text{if} \ \hat{r} < \hat{r}_R \ \text{and} \ 0 < \hat{q} < \hat{q}_R \\
0, & \text{if otherwise}
\end{cases}
\end{align*}
\]

where \( \hat{q}_R = \frac{2q}{\gamma + 2\lambda} \) is the upper bound of \( q \) for \( \hat{r}_R > \hat{r} \), and

\[
\begin{align*}
\hat{\eta}_R^2 &= \frac{1}{2 \lambda \hat{q}^2 X^2} \cdot \left\{ 3 + 3 \lambda \hat{q} X \sigma^2 \left[ \gamma (2 - \hat{q}) - 2 \lambda \hat{q} \right] - \sqrt{9 + 6 \lambda \hat{q} X^2 \sigma^2 \left[ \gamma (2 - \hat{q}) - 2 \lambda \hat{q} \right]} \right\} \\
\hat{\hat{r}}_R &= \hat{r} + \frac{1}{\lambda \hat{q} X} \cdot \left\{ \sqrt{9 + 6 \lambda \hat{q} X^2 \sigma^2 \left[ \gamma (2 - \hat{q}) - 2 \lambda \hat{q} \right]} - 3 \right\}
\end{align*}
\]

The government chooses \( \hat{q}^{ast} \) to maximize expected social welfare, implicitly determined by

\[
\sqrt{9 + 6 \lambda \hat{q} X^2 \sigma^2 \left[ \gamma (2 - \hat{q}) - 2 \lambda \hat{q} \right]} \cdot \left\{ 3 + 3 \lambda \hat{q} X \sigma^2 \left[ \gamma (2 - \hat{q}) - 2 \lambda \hat{q} \right] - \sqrt{9 + 6 \lambda \hat{q} X^2 \sigma^2 \left[ \gamma (2 - \hat{q}) - 2 \lambda \hat{q} \right]} \right\}
\]

where \( \hat{q} < \frac{2q}{\gamma + 2\lambda} \). \( \hat{q}^{ast} \) must be a function of \( \lambda \) and \( \gamma \), and Proposition 5.2 follows naturally.

**Proposition 5.2** There is no optimal flat retention ratio requirement for all markets with securitizers and investors of different risk attitude.

If the regulatory authority does not directly stipulates \( \hat{q} \), but rather requires the originating bank to disclose its retention ratio, or equivalently, \( q \) to the market, then \( q \) can be taken by purchasers as a noiseless signal to infer the quality of underlying assets in the separating equilibrium, which is proved by DeMarzo and Duffie (1999) that when \( \hat{r} \) has a finite support (as in our model), there is a separating equilibrium that Pareto dominates other equilibria under weak \( D_1 \) refinement criterion of Cho and Kreps (1987).

\[
\hat{r}(q_D(\hat{r})) = \hat{r}
\]

where \( q_D \) is the optimal securitization intensity chosen by the bank. This is defined as “equilibrium market valuation schedule” in Leland and Pyle (1977) to stress the property that the market can correctly identify the true quality \( \hat{r} \) with an equilibrium market valuation schedule. Conjecture a monotonically decreasing market valuation scheme \( \hat{p}(q) = E(\hat{r}|q) = \hat{r}(q) \), with \( \hat{r}'(q) < 0 \).
Proposition 5.3 The equilibrium asset price and securitization intensity in the market with D Regulation are:

\[ p_D = \hat{r} - \lambda \sigma^2 q_D X \]  

and \( q_D \) is the inverse function of \( \hat{r}(q_D) \),

\[ \hat{r}(q_D) = \gamma \sigma^2 X \cdot (q_D - \ln q_D) + 2 \lambda \sigma^2 X q_D + X \cdot (q_D - \ln X) + 2 \lambda \sigma^2 X q_D + \hat{r} - \gamma \sigma^2 X (1 - \ln \frac{\gamma}{\gamma + 2\lambda}) \]  

As expected, \( q \) is read by the market investors as a negative signal of the underlying asset quality.

Note that \( q_D(\bar{r}) \) is strictly positive by observing

\[ \bar{r} - \underline{r} + \gamma \sigma^2 X (1 - \ln \frac{\gamma}{\gamma + 2\lambda}) = \gamma \sigma^2 X \cdot (q_D - \ln q_D) + 2 \lambda \sigma^2 X \cdot q_D \]

the LHS of which is strictly positive and finite, so should be its RHS. Since \( \lim_{q \to 0^+} \ln q = -\infty \), it follows that \( q_D(\bar{r}) \) is strictly positive.

5.2. Evaluating Welfare Effect of R Regulation and D Regulation

It is straightforward to notice that structural distortion can be corrected with either R Regulation or D Regulation:

\[ \int_{\underline{r}}^{\bar{r}} q X [\hat{r} - \hat{r}(\Omega)] d\hat{r} = 0 \]

Clearly, D Regulation also corrects the level distortion: \( AS_D = 0 \).

Proposition 5.4 R Regulation is effective in correcting structural distortion resulted from asset opaque-ness; D Regulation is effective in correcting both structural and level distortion.

However, observing Figures 3-1 and 3-2, it follows that correcting structural distortion creates a new distortion: R Regulation aggravates level distortion, while D Regulation is costly since the bank has to distort its securitization distortion away from the efficient level so as to send signals to the market, although both level distortion and structural distortion can be corrected with D Regulation.

Correcting level and structural distortion by D Regulation unanimously increases social welfare, and the more so if the adverse selection problems in the unregulated or R regulated cases are more severe, i.e. if \( \gamma \) decreases, or \( \lambda \) increases. However, it is found by observing Figure 3-2 that the regulatory cost of D Regulation is the signalling cost: in order to signal asset quality, the originating bank has to distort securitization intensity of relatively high quality assets, to a greater extent than the unregulated case.

Concerning R Regulation, although it corrects structural distortion, it is costly because it worsens level distortion. Formally, level distortion with R Regulation is \( LD \equiv AS_R = \bar{r} - \hat{r}_R \).
Lemma 5.1  
Adverse selection problem with R Regulation is more severe if the bank is less risk averse, or if the investors are more risk averse, or if the stipulated \( q \) is higher.

Intuitively, when \( \lambda \) increases, the price that the market is willing to pay is lower, forcing the bank to sell lower quality asset, which further lowers the average asset quality and aggravate the lemons effect. An increase in \( \overline{q} \) is read by the market to reflect worse asset quality, leading to a more pronounced adverse selection problem.

Proposition 5.5  
The level distortion resulted from informational asymmetry is aggravated by R Regulation, and this aggravation is more prominent if \( \gamma \) or \( \lambda \) increases.

Intuitively, the aggravated adverse selection problem in this case is due to the “information destruction effect” of R Regulation. Comparing with the unregulated scenario, it seems that market investors have available additional information \( \overline{q} \), but actually not: \( q = \overline{q} > 0 \) is less informative than \( q > 0 \) in the unregulated case, since \( \overline{q} \) does not reflect the optimal choice of banks and thus performs worse in reflecting the quality of underlying assets than \( q > 0 \) in the unregulated case.

In summary, comparing with the unregulated scenario, level distortion of R Regulation is more aggravated if \( \gamma \) or \( \lambda \) increases, which decreases the social welfare. Structural distortion of R Regulation is mitigated if \( \gamma \) increases or \( \lambda \) decreases. Therefore, only when \( \gamma \) is relatively high is R Regulation possible to increase expected social welfare; the effect of \( \lambda \) is indefinite. Unfortunately, no further analytical results can be derived and we illustrate the welfare effect of regulation with a numerical example.

5.3. A Numerical Example

Assume \( \sigma^2 = 1, X = 1 \). \( \overline{q}^* \) implicitly determined by (5.5) is depicted by the bold line in Figures 4-1 and 4-2. Outside the range as depicted, there is no such \( \lambda \) and \( \gamma \) that lead to \( \overline{q}^* = 0.95 \) or \( \overline{q}^* = 0.5 \).

![Figure 4-1. \( \overline{q}^* = 0.95 \) but Market Breaks Down](image1)

![Figure 4-2. \( \overline{q}^* = 0.5 \) and No Market Breakdown](image2)

In these figures, the line for the market not to completely break down, \( \overline{q} < \frac{2\gamma}{\gamma + 2\lambda} \), is also depicted. From Figure 4-1, the stipulation of \( \overline{q} = 0.95 \) will result in a complete market breakdown, while not the case if requires \( \overline{q} = 0.5 \). Although the specific value of optimal \( \overline{q} \) depends on our model specifications, uniform distribution for instance, it implies that the regulator should be careful in stipulating \( \overline{q} \). Moreover, there is no optimal flat rate of retention ratio requirement for any combination of \( \lambda \) and \( \gamma \); either \( \overline{q}^* = 0.95 \) or \( \overline{q}^* = 0.5 \) is optimal only with measure 0.

Therefore, in what follows, we examines the welfare effect of flexible \( \overline{q} \) regulation for different \( \lambda \) and \( \gamma \), taking \( \lambda = 0.5, 1, 3, 5 \) or \( \gamma = 1, 3, 5, 10 \) for illustration (Figures 5-1 and 5-2).
First examine when R Regulation improves social welfare, i.e. regulatory cost is outweighed by the corrected structural distortion (Figure 6).

Observing Figure 6, R Regulation improves social welfare upon the unregulated case when $\gamma$ is relatively large, which mainly because in this case structural distortion is high if unregulated. The welfare effect with respect to $\lambda$ is indeterminant.

As is clear from the above analysis, the essence of R Regulation is to correct one distortion by worsening the other one. If the regulatory authority’s objective is simply to correct structural distortion (maybe to reduce systemic risk), R Regulation is indeed effective. However, it worsens the informational asymmetry and is passive in the sense that it does not aim to dissolve the source of the original distortion, but simply scratch the surface by imposing new distortions. Figure 7 shows whether D Regulation can complement R Regulation under some conditions.

From Figure 7, D Regulation complements R Regulation when $\lambda$ is relatively large. This is mainly because in this case adverse selection is more prominently aggravated by R Regulation. When $\lambda$ is relatively small, D Regulation is better than R Regulation only when $\gamma$ is fairly small.

The finding that D Regulation is better than R Regulation if investors are relatively conservative may imply R Regulation is more effective for junior tranches since investors in junior tranches are generally more risk seeking, and thus our findings basically proclaim for the first-loss piece retention mechanism.
6. CONCLUSION

In this model, we build up a simple two period model to evaluate two recently proposals to regulate bank asset securitization activities in United States and the European Union: retention ratio regulation and information disclosure regulation. In a world where a risk averse bank securitizes opaque assets and sells to a continuum of uninformed risk averse market investors. Due to the familiar lemons effect, market investors discount asset price to be on average lower than its true value. This distorts bank’s securitization incentive as well as the welfare allocation between the seller and buyers, represented by level and structural distortions in our model, respectively.

We find that it is impossible for a flat rate retention ratio requirement to be optimal for all markets. Although both regulatory proposals are effective in reducing investors’ informational loss, neither can improve upon the unregulated case for all parameters, due to the associated regulatory cost: retention ratio regulation aggravates adverse selection problem due to “information destruction effect”, and information disclosure requirement leads to a distortion in securitization intensity so as to send signals. However, information disclosure regulation complements retention ratio regulation when investors are relatively risk averse.

Some of our findings quantitatively rest on the assumption that loan assets are uniformly distributed. This simplifies our analysis significantly, but it will be illuminating if similar findings can be analytically shown to qualitatively hold in a much more general framework.
APPENDIX A: PROOF OF PROPOSITIONS

A.1. Proof of Proposition 3.1

Proof: From (3.1),

\[ y_t^* = \frac{E(\hat{\delta}(\Omega)) - p}{\lambda \cdot [Var(\hat{\delta}(\Omega)) + \sigma^2]} \]

Market clearing condition requires \( \int_0^1 y_t^* \cdot q \cdot d \lambda = q \cdot X \), and therefore,

\[ p(q) = E(\hat{\delta}(\Omega) - \lambda q X \cdot [Var(\hat{\delta}(\Omega)) + \sigma^2]) \]

Without information asymmetry, replace \( E(\hat{\delta}(\Omega)) = \hat{p} \) and \( Var(\hat{\delta}(\Omega)) = 0 \) in (A.2) gets the market price in the benchmark case:

\[ p_B(q) = \hat{p} - \lambda q X \cdot \sigma^2 \]

inserting which into (3.2) and from the first order condition,

\[ p_B = \hat{p} - \lambda q \sigma^2 X \cdot q_B, \quad q_B = \frac{\gamma}{\gamma + 2\lambda} \]

Q.E.D.

A.2. Proof of Proposition 4.1

Proof: Although securitization quantity \( q \) is not revealed to the market and thus investors cannot infer asset information from the specific value of \( q \), the coarse information whether \( q > 0 \) or not is still available to investors.

We only prove the case when \( \gamma < 2\lambda(1 + \frac{1}{4}\gamma^2\sigma^2 X^2) \). The other case follows naturally. At the initial round of logical induction,

\[ p(0) = E(\hat{\delta}) - \lambda q(0) X \cdot (\eta^2 + \sigma^2) \]

Therefore, the bank’s problem is:

\[ \max_{q} \quad Kr_f + D(r_f - r_D) + qX \cdot [E(\hat{\delta}) - r_f] + (1 - q)X(\hat{p} - r_f) - \frac{1}{2}\gamma \sigma^2 X^2 \cdot (1 - q)^2 - \lambda q^2 X^2 \cdot (\eta^2 + \sigma^2) \]

from which

\[ q(0) = \begin{cases} \frac{E(\hat{\delta}) - \hat{p} + \gamma \sigma^2 X}{2\lambda X (\eta^2 + \sigma^2) + \gamma \sigma^2 X}, & \text{if } \hat{p} < E(\hat{\delta}) + \gamma \sigma^2 X \\ 0, & \text{if } \hat{p} > E(\hat{\delta}) + \gamma \sigma^2 X \end{cases} \]

If the market investors observe \( q(0) > 0 \), the price cannot be (A.5); the price would rather be

\[ p(1) = E(\hat{\delta}) \leq \hat{p} < E(\hat{\delta}) + \gamma \sigma^2 X - \lambda q(1) X \cdot (\eta_1^2 + \sigma^2) = \frac{\hat{p} + \gamma \sigma^2 X - \lambda q(1) X \cdot (\eta_1^2 + \sigma^2)}{2} \]

where \( \eta_1^2 \equiv \frac{(E(\hat{\delta}) + \gamma \sigma^2 X - \hat{p})^2}{12} \).

Under Assumption A, \( E(\hat{\delta}) + \gamma \sigma^2 X < \hat{p} \) and lemons’ effect is involved. It is straightforward to show that there is always an adverse selection effect in the following rounds if \( E(\hat{\delta}) + \gamma \sigma^2 X > 0 \), which holds unconditionally.

\[ q(1) = \begin{cases} \frac{E(\hat{\delta}) - \hat{p} + \gamma \sigma^2 X}{2\lambda X (\eta_1^2 + \sigma^2) + \gamma \sigma^2 X}, & \text{if } \hat{p} < E(\hat{\delta}) + \gamma \sigma^2 X - \lambda q(2) X \cdot (\eta_2^2 + \sigma^2) \\ 0, & \text{if } \hat{p} > E(\hat{\delta}) + \gamma \sigma^2 X - \lambda q(1) X \cdot (\eta_1^2 + \sigma^2) \end{cases} \]

Similarly, the \( n^{th} \) round of deduction generates the price as follows:

\[ p(n) = \frac{\hat{p}}{2n+1} \cdot \hat{p} + \left( \frac{1}{2} + \frac{1}{2^2} + \cdots + \frac{1}{2^n} \right) \cdot \left( E(\hat{\delta}) + \gamma \sigma^2 X \right) - \lambda q(n) X \cdot (\eta_n^2 + \sigma^2) \]

In the limit,

\[ p_U = E(\hat{\delta}) + \gamma \sigma^2 X - \lambda q_U X \cdot (\eta_U^2 + \sigma^2), \quad q_U = \begin{cases} \frac{\hat{p} + \gamma \sigma^2 X - \lambda q_U X \cdot (\eta_U^2 + \sigma^2)}{2XX (\eta_U^2 + \sigma^2) + \gamma \sigma^2 X}, & \text{if } \hat{p} < \hat{p} U \\ 0, & \text{if } \hat{p} > \hat{p} U \end{cases} \]

where \( \eta_U^2 \equiv \frac{(\hat{p} + \gamma \sigma^2 X - \hat{p})^2}{12} = \frac{1}{4} \gamma^2 \sigma^2 X^2 \). It is not difficult to verify that \( p_U > 0 \) for all \( \hat{p} \) (which suffices to verify \( p_U > 0 \) when \( \hat{p} = \hat{p} \)).

To verify that \( p_U \) is indeed the equilibrium price, imagine there is a next round where

\[ p = E(\hat{\delta} \hat{p}) \quad \text{if } \hat{p} < \hat{p} \quad \text{and} \quad \hat{p} + 2\gamma \sigma^2 X - \lambda q_U \cdot (\eta_U^2 + \sigma^2) \]

which shows the stationarity of the equilibrium price.\(^8\) Q.E.D.

\(^8\)Surely, this stationary property can be used to solve for the equilibrium price, but the logical deduction procedures better exhibits the adverse selection effect involved.
A.3. Proof of Lemma 4.1

Proof: Replacing the expressions of \( \hat{r}_U \),
\[
\frac{\partial AS_U}{\partial \gamma} = -2\sigma^2 X < 0
\]
Q.E.D.

A.4. Proof of Proposition 4.2

Proof: This is equivalent to prove \( \int q_U X [\hat{r} - E(\hat{r}|q_U) > 0] \) \( d\hat{r} < 0 \). If \( \gamma < 2\lambda(1 + \frac{1}{3}\gamma^2\sigma^2X^2) \),
\[
\int q_U X [\hat{r} - E(\hat{r}|q_U) > 0] \) \( d\hat{r} = \int_{\hat{r}}^{\infty} \frac{X + 2\sigma^2 X - \hat{r}}{2\lambda X \cdot (\eta_U^2 + \sigma^2) + \gamma\sigma^2 X} \cdot X \cdot \frac{X - \gamma\sigma^2 X}{d\hat{r}} \]
\[
= -\frac{2(\gamma\sigma^2 X)^3}{3(2\lambda(\eta_U^2 + \sigma^2) + \gamma\sigma^2)} < 0
\]
If \( \gamma > 2\lambda(1 + \frac{1}{3}\gamma^2\sigma^2X^2) \),
\[
\int q_U X [\hat{r} - E(\hat{r}|q_U) > 0] \) \( d\hat{r} = -\frac{1}{6} X^3 \cdot [\gamma\sigma^2 - 4\lambda^2(\eta_U^2 + \sigma^2)^2] < 0
\]
Q.E.D.

A.5. Proof of Proposition 5.1

Proof: At the initial round of logic deduction, investors cannot infer anything about the asset quality from a flat rate of \( \bar{q} \).
\[
p^{(0)} = E(\hat{r}) - \lambda qX \cdot (\sigma^2 + \eta^2)
\]
Since \( \bar{q} \) is imposed by the regulator, and thus the bank cannot optimize to decides the securitization intensity. Thus the bank would prefer to securitize \( \bar{q} \) proportion of its asset at price \( p^{(0)} \) if and only if \( EU_{bank}(q = \bar{q}) > EU_{bank}(q = 0) \):
\[
Kr + D(r_f - r_D) + \bar{q} X \cdot (p^{(0)} - r_f) + (1 - \bar{q}) X \cdot (\hat{r} - r_f) - \frac{1}{2} \gamma\sigma^2 X^2 \cdot (1 - \bar{q})^2
\]
\[
> Kr + D(r_f - r_D) + X \cdot (\hat{r} - r_f) - \frac{1}{2} \gamma\sigma^2 X^2
\]
from which
\[
\hat{r}^{(0)} < E(\hat{r}) + \gamma\sigma^2 X - \frac{1}{2} \gamma\sigma^2 X \bar{q} - \lambda \bar{q} X \cdot (\sigma^2 + \eta^2) < \bar{r}
\]
Similarly, when the uninformed investors observe \( q = \bar{q} \neq 0 \), the price of securitized asset cannot be \( p^{(0)} \), and the \( n \)th round of logical deduction generates the following condition:
\[
\hat{r}^{(n)} < (\frac{1}{2} + \frac{1}{2^n + 1}) \cdot \left[ \bar{q} + \gamma\sigma^2 X (2 - \bar{q}) - 2\bar{q}\lambda X \sigma^2 \right] + \frac{1}{2^n + 1} \cdot \hat{r} - \lambda \bar{q} X \cdot \lim_{n \to +\infty} \sum_{i=0}^{n} (\eta_i^2 \cdot \frac{1}{2^{n-i}})
\]
Therefore, after \( n \) rounds,
\[
\hat{r}^{(n)} < \bar{q} + \gamma\sigma^2 X (2 - \bar{q}) - 2\bar{q}\lambda X \sigma^2 - \lambda \bar{q} X \cdot \lim_{n \to +\infty} \sum_{i=0}^{n} (\eta_i^2 \cdot \frac{1}{2^{n-i}})
\]
In the limit, the variance \( \eta_R^2 \) is stationary at \( \eta_R^2 \), and thus
\[
\lim_{n \to +\infty} \sum_{i=0}^{n} (\eta_i^2 \cdot \frac{1}{2^{n-i}}) = 2\eta_R^2
\]
Inserting which into (A.19), the bank securitizes \( \bar{q} \) proportion of its assets with quality
\[
\hat{r} < \hat{r}_R \equiv \bar{q} + \gamma\sigma^2 X (2 - \bar{q}) - 2\bar{q}\lambda X (\sigma^2 + \eta_R^2)
\]
where \( \eta_R^2 \) is determined by
\[
\eta_R^2 = \frac{[\gamma\sigma^2 X (2 - \bar{q}) - 2\bar{q}\lambda X (\sigma^2 + \eta_R^2)]^2}{12}
\]
which reduces to (5.3).
For the asset securitization market not to completely break down by the aggrandized lemon’s effect, it requires \( \hat{r}_R > \bar{r} \), or equivalently, \( 0 < \bar{q} < \frac{\bar{r}}{\gamma^2} \). Therefore, \( q_R = \bar{q} \) only when \( 0 < \bar{q} < \bar{q}_R \) and \( \bar{r} < \hat{r}_R \) and the equilibrium price and quantity of securitized assets are as summarized in the proposition.
Q.E.D.
A.6. Proof of Lemma 5.1

**Proof:** Replacing the expressions of \( \hat{r}_R \),

\[
\frac{\partial AS_R}{\partial \gamma} = -\frac{3\sigma^2 X (2 - \hat{q})}{\sqrt{9 + 6\hat{q}^2 X^2 \sigma^2 - \left[\gamma (2 - \hat{q}) - 2\lambda \hat{q}\right]}} < 0
\]

\[
\frac{\partial AS_R}{\partial \lambda} = \frac{3 \left[ 3 + 3\lambda \hat{q} X^2 \sigma^2 - 2 \sqrt{9 + 6\hat{q}^2 X^2 \sigma^2 - \left[\gamma (2 - \hat{q}) - 2\lambda \hat{q}\right]} \right]}{\lambda \hat{q}^2 X \cdot \sqrt{9 + 6\hat{q}^2 X^2 \sigma^2 - \left[\gamma (2 - \hat{q}) - 2\lambda \hat{q}\right]}} > 0
\]

\[
\frac{\partial AS_R}{\partial \hat{q}} = \frac{3 \left[ 3 + 2\lambda \hat{q} X^2 \sigma^2 - 2 \sqrt{9 + 6\hat{q}^2 X^2 \sigma^2 - \left[\gamma (2 - \hat{q}) - 2\lambda \hat{q}\right]} \right]}{\lambda \hat{q}^2 X \cdot \sqrt{9 + 6\hat{q}^2 X^2 \sigma^2 - \left[\gamma (2 - \hat{q}) - 2\lambda \hat{q}\right]}} > 0
\]

Q.E.D.

A.7. Proof of Proposition 5.3

**Proof:** Conjecture a monotonically decreasing market valuation scheme \( p(q) = E(\hat{r}|q) = \hat{r}(q) \), with \( \hat{r}'(q) < 0 \). Thus in the separating equilibrium, \( E(\hat{r}|q) = \hat{r}(q) \) and \( \text{Var}(\hat{r}|q) = 0 \), transforming the equilibrium price (A.2) into:

\[
p(q) = \hat{r}(q) - \lambda \sigma^2 q X
\]

Then the bank’s problem is

\[
\max_q \quad K r_f + D(r_f - r_D) + q X \cdot [\hat{r}(q) - r_f] + (1 - q) X (\hat{r} - r_f) - \frac{1}{2} \gamma \sigma^2 X^2 \cdot (1 - q)^2 - \lambda \sigma^2 q X^2
\]

the first order condition of which

\[
\hat{r}(q) - \hat{r} + q \cdot \hat{r}'(q) + q \sigma^2 X (1 - q) - 2 \lambda \sigma^2 q X = 0
\]

Imposing (5.6) on (A.28),

\[
q \cdot \hat{r}'(q) + \gamma \sigma^2 X (1 - q) - 2 \lambda \sigma^2 q X = 0
\]

Solving this differential equation,

\[
\hat{r}(q_D) = \gamma \sigma^2 X \cdot (q_D - \ln q_D) + 2 \lambda \sigma^2 X q_D + C
\]

where \( C \) is the differential constant. It can be verified that \( \hat{r}'(q_D) < 0 \) if and only if \( 0 < q_D < \frac{\gamma \sigma^2 X}{\lambda + 2\lambda} \), which holds by observing (A.29).

Equilibrium \( q_D(\hat{r}) \) is the inverse equation of \( \hat{r}(q_D) \), and \( p_D = \hat{r}(q_D) - \lambda \sigma^2 q_D X \). It can be straightforward to verify that the bank strictly prefers to securitize \( q \) at \( p \) to not securitize at all (the condition of which actually reduces to an unconditionally holding inequality: \( q < 1 \)).

From (5.6), it is known that \( \hat{r}'(q) \cdot q_D(\hat{r}) = 1 \),

\[
\hat{r}'(q_D) < 0, \quad \hat{r}''(q_D) > 0, \quad \therefore \quad q_D'(\hat{r}) > 0, \quad q_D''(\hat{r}) > 0
\]

Therefore, \( q_D(\hat{r}) \) is decreasing and convex in \( \hat{r} \), and \( \max_{q \in [0, 1]} q_D(\hat{r}) = q_D(\zeta) \). Thus without loss of generality, assume \( q_D(\zeta) = \frac{\zeta}{\gamma + 2\zeta} \), and the differential constant \( C = \frac{2}{\gamma + 2\zeta} \). Q.E.D.

A.8. Proof of Proposition 5.5

**Proof:** Replacing the expressions of \( \hat{r}_R \),

\[
\frac{\partial AS_R - AS_U}{\partial \gamma} = \frac{1}{\lambda \hat{q} X} \cdot \left\{ 3 + 2\lambda \hat{q} X^2 \sigma^2 - \sqrt{9 + 6\hat{q}^2 X^2 \sigma^2 - \left[\gamma (2 - \hat{q}) - 2\lambda \hat{q}\right]} \right\} > 0
\]

\[
\frac{\partial AS_R - AS_U}{\partial \lambda} = \sigma^2 X \cdot \left\{ 2 - \frac{3(2 - \hat{q})}{\sqrt{9 + 6\hat{q}^2 X^2 \sigma^2 - \left[\gamma (2 - \hat{q}) - 2\lambda \hat{q}\right]} \right\} > 0
\]

\[
\frac{\partial AS_R - AS_U}{\partial \hat{q}} = \frac{3 \left[ 3 + 3\lambda \hat{q} X^2 \sigma^2 - 2 \sqrt{9 + 6\hat{q}^2 X^2 \sigma^2 - \left[\gamma (2 - \hat{q}) - 2\lambda \hat{q}\right]} \right]}{\lambda \hat{q}^2 X \cdot \sqrt{9 + 6\hat{q}^2 X^2 \sigma^2 - \left[\gamma (2 - \hat{q}) - 2\lambda \hat{q}\right]}} > 0
\]

Q.E.D.
REFERENCES


