Pricing Convertible Bonds with Embedded Parisian Options: Theory and Evidence

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Abstract

We propose and empirically investigate a two-factor pricing model for convertible bonds with embedded Parisian options (soft call protection) and stochastic interest rate. The model is solved numerically by a finite element method. Studying the 49 convertible bonds and 47 months of weekly market prices in China, we find that there is no significant mispricing on average, i.e. the market prices are almost equal to our model prices. Ignoring the embedded Parisian options, however, will dramatically decrease the model prices such that the market prices are overpriced by 5.61%. Our result shows that the Parisian options have a significant effect on pricing convertible bonds in the markets where soft call protection is prevailing.

Key Words: Convertible bonds, pricing, Parisian options
JEL Codes: G12, G13, G15

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1. Introduction

Convertible bonds are hybrid instruments for raising capital on financial markets. They offer investors the right to give up a bond in exchange to a specified number of shares of common stock. In 2007, total convertible bond issuance in the US was $56 billion\(^1\) while the initial equity offering in the same year was $35 billion\(^2\). Despite the significant influence on financial markets, pricing convertible bonds remains a challenge due to the complex embedded options, i.e., convertible bonds normally allow the bond issuer (holder) to call (sell) back the bonds at a pre-decided call (put) price only if the underlying stock price is above (below) the trigger price for a certain prescribed and consecutive time. Such options are named Parisian options. As a hybrid product with equity and fixed income characteristics, convertible bonds are also subject to default risk, interest rate risk, and market risk of stock price.

This paper is therefore motivated to answer two questions. First, how can we solve the pricing model that incorporates the possibility of early conversion, callability by the issuer and putability by the holder with Parisian features, stochastic interest rate, and credit risk? Second, do Parisian options matter when we empirically price the convertible bond?

We propose a contingent claim model with two factors: stock price and interest rate. Following Barone-Adesi et al. (2003), we value a convertible bond with Parisian options by a finite element method. This method gives convergent deterministic approximations under realistic and low smoothness assumptions on the payoff function and in particular allows a higher rate of convergence compared with finite difference method. We model the underlying stock process under Black-Scholes Geometric Brownian Motion (GBM) framework and

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\(^1\) See Choi, Getmansky, Henderson, and Tookes (2010).
stochastic interest rates process as Cox–Ingersoll–Ross (CIR). In addition, we allow an instantaneous correlation between the dynamics of stock and interest rate. A new state variable $\tau$ is used to model the barrier time for a Parisian option following Haber et al. (1999). Pricing partial differential equations (PDE) are then derived and solved numerically.

This paper contributes to the literature in the following aspects. First, we solve the theoretical price for convertible bonds with embedded Parisian options and stochastic interest rate in a stock-based model. The literature on the theoretical modeling of convertible bond prices can be divided into two branches based on the underlying asset: firm value or stock. Ingersoll (1977) finds a closed-form solution to the convertible bond price depending on the firm value as the underlying state variable. The pricing formula is further developed by Lewis (1991) and Buehler and Koziol (2002) accounting for more complex capital structures. The first numerical solution for the firm-value-based models is found in Brennan and Schwartz (1977). Brennan and Schwartz (1980) extend their pricing method by including stochastic interest rate. Their model is further extended by Buchan (1998) to allow senior debt. Because the Parisian options are written on the stock price and firm values are not observable, firm value models have difficulties in modeling these path-dependent contingent claims.

Stock-based models are more flexible to handle complexities in convertible bonds. These models were initiated by McConnell and Schwartz (1986). To account for credit risk, they use a constant credit spread. Bardhan et al. (1993) and Tsiveriotis and Fernandes (1998) propose an approach that splits the value of a convertible bond into a stock component and a straight bond component so that the credit risk of a convertible bond can vary across its moneyness. Ammann et al. (2003) extend this approach by accounting for call features with various trigger conditions. A particular credit-risk approach following the reduced-form Duffie and Singleton (1999) model is applied by Davis and Lishka (1999), Takahashi et al.
Hung and Wang (2002), Carayannopoulos and Kailimipalli (2003), and Ayache et al. (2003). In all these models, the Parisian features that require the stock price to stay above the trigger price for a consecutive period are not explicitly considered.

Our paper is most close to the simulation-based pricing model by Ammann et al. (2008) in that we all consider the embedded Parisian options, the possibility of early conversion, and exogenous credit risk. However, we keep our model in the classical framework of partial differential equation approach so that we can avoid the computational burden and potential estimation error from simulation. Moreover, our paper also includes a stochastic interest rate process. Though the effect of a stochastic term structure on convertible bond prices is rather small as is shown in this paper, our two-factor model provides a theoretical foundation of handling two state variables with Parisian features.

The second contribution of this paper is to quantify the effect of Parisian options and stochastic interest rate by calibrating the model to the Chinese market. We compute the mean pricing error for different variant of our benchmark model by turning on and off the Parisian feature or the stochastic feature of the interest rate. The result shows that the Parisian options account for more than 5% of the convertible bond price on average. This is a rather large effect given that the mean pricing error in the literature is reported to be between -12.9% and 1.7%. In contrast, the effect of the stochastic interest rate is only 0.2%. This small effect is consistent to the findings in Brennan and Schwartz (1980) and Ammann et al. (2008).

The third contribution of this paper is an empirical analysis of the Chinese convertible bond market. As one of the most important emerging markets, China-based IPOs led the world both in number of deals and dollar volume raised in 2009. Convertible bonds, although the current market capitalization cannot be comparable to the domestic equity

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3 See King (1986), Carayannopoulos (1996), Buchan (1997), and Ammann et al. (2003).
market yet\(^5\), are expected to have a tremendous growth soon given the huge demand of capital from Chinese firms. For example, the two biggest banks in China have announced their plans to sell convertible bonds as much as $9.56 billion in total\(^6\), almost twice the size of the current convertible bond market. However, very little empirical research has been undertaken for the Chinese convertible bond market. Previous empirical research mainly focused on developed markets. King (1986) examines a sample of 103 American convertible bonds and finds that market prices are 3.75% below model prices (underpricing). Using monthly price data, Carayannopoulos (1996) empirically investigates 30 American convertible bonds for a one-year period and finds a larger mean underpricing of 12.9%. In contrast, Buchan (1997) finds that model prices for 35 Japanese convertible bonds are slightly below observed market prices on average by 1.7% (overpricing). Carayannopoulos and Kalimipalli (2003) study 25 US convertible bonds. Ammann et al. (2003) investigate the daily prices for 21 French convertible bonds and find an average underpricing of 3.24%. Ammann et al. (2008) investigate 32 US convertible bonds with a pricing model based on Monte Carlo estimation that account for Parisian features. They find a slight overpricing of 0.36% with a standard deviation of 6.17%.

Using a partial differential equation method, we find that the market prices are almost equal to our model prices with a slight underpricing of 0.001%. Our result is in line with the recent finding by Ammann et al. (2008) that the long-standing underpricing puzzle of convertible bonds is not supported by the recent data. Our sample for Chinese convertible bonds uses 47 months of weekly data, ranging from Mar 2, 2006, to Feb 1, 2010.

The structure of the paper is as follows. In Section 2 we present our two-factor pricing model with Parisian options for convertible bonds and the numerical methodology for

\(^5\) As of April of 2009, capitalization of about 20 convertible bonds traded on the Shanghai and Shenzhen stock exchanges was $4.8 billion. “China Convertible Bond Market Flourishes in Crisis”, Reuters, April 3, 2009.

solving it. In Section 3 we describe the data set and the input parameters. In Section 4 we present the empirical results. Section 5 concludes.

2. The pricing model

In this section, we develop our theoretical model for Parisian callable and puttable convertible bonds by monitoring the Parisian options with a state variable $\tau$. We derive our pricing partial differential equations (PDE) under no-arbitrage conditions. We also provide a variational formulation to solve the PDE with the finite element method.

2.1. The theoretical model

We start our model with the dynamics of two basic factors: equity and interest rate. They are given by the following diffusion processes:

$$dS = (\mu - d)dt + \sigma S d\omega_1$$ \hspace{1cm} (1)

$$dr = u(r,t)dt + \sigma(r,t)d\omega_2$$ \hspace{1cm} (2)

where $S$ is the underlying stock price, $r$ is the spot interest rate, $\omega_1$ and $\omega_2$ are Wiener processes with correlation $\rho$, $\mu$ is the expected rate of return, $d$ is the dividend rate\(^7\), and $\sigma$ is the volatility of the underlying stock. $u$ and $\sigma$ are the expected rate of return and volatility for the interest rate process and they can be time dependent.

Besides the dynamics of equity and interest rate, following Haber et al. (1999), we introduce a new state variable $\tau$ to monitor the barrier time for Parisian options. The barrier time $\tau$ is defined as the length of time that the underlying stock price has been above an upper barrier or below a lower barrier in the current excursion. Taking an upper barrier as an example,

\(^7\) When the underlying stock pays dividends, the strike price of convertible bonds in Chinese market will be adjusted proportionally, this adjustment “eliminates” the influence of future dividends on the price of convertible bonds, therefore we let $d = 0$ for pricing equations derivation and empirical research.
\[
\tau := t - \sup \{s \leq t \mid S_s \leq L \}
\]

where \(s\) is the last time the underlying stock price was below the barrier price \(L\). Therefore \(\tau\) measures the difference between the current time \(t\) and \(s\). The dynamic process of \(\tau\) is governed by

\[
d\tau(t) = \begin{cases} 
  dt & \text{if } S(t) > L \\
  -\tau(t-) & \text{if } S(t) = L \\
  0 & \text{if } S(t) < L 
\end{cases}
\]

where \(\tau(t-)\) is the left limit of \(\tau\). The intuition behind this process is that \(\tau\) and \(t\) change at the same rate when the stock price is above the barrier; \(\tau\) is reset to zero when the stock price is equal to the barrier; and \(\tau\) does not change when the stock price is below the barrier.

Let \(V\) be the price of a convertible bond with maturity date \(T\). The value of a convertible bond becomes a measurable function \(V = V(S, r, t, \tau; T)\) with \(T > t\). With the definition of those dynamic processes from (1) to (4), we are able to construct a portfolio \(\Pi\) consisting of one convertible bond \(V\), \(-\Delta\) shares of another convertible bond \(V_1\) and \(-\Delta\) shares of the underlying stock \(S\). Thus,

\[
d\Pi = dV - \Delta_1 dV_1 - \Delta dS.
\]

By Ito’s lemma and under no-arbitrage conditions, it can be shown (see Wilmott et al. (1993)) that the fair value of a convertible bond satisfies the following PDE\(^8\):

\[
\frac{\partial V}{\partial t} + \frac{\partial V}{\partial \tau} dt + \frac{1}{2} \sigma^2 S^2 \frac{\partial^2 V}{\partial S^2} + \frac{1}{2} \sigma^2 \frac{\partial^2 V}{\partial r^2} + \rho \sigma \sigma \frac{\partial^2 V}{\partial S \partial r} + \frac{\partial^2 V}{\partial \tau^2} - rV + rS \frac{\partial V}{\partial S} + K(t) = -(u - \lambda \sigma) \frac{\partial V}{\partial r},
\]

where \(K(t)\) is the coupon payment at \(t\), and \(\lambda\) is the market price of interest rate risk.

Considering the impact of credit spread on the price of a convertible bond, Tsiveriotis and Fernandes (1998) value a convertible bond with credit risk by separating it into two

\(^8\) We suppress the time dependence to keep the notation light.
securities: a cash-only part (COCB) and an equity-like part (ELCB). The holder of a COCB is entitled to all cash flows but no equity flows. Therefore the COCB is subject to default and discounted at a risky rate (risk free rate plus credit spread \( r_c \)). On the other hand, the ELCB is default-free and discounted at a risk free rate. Since both parts are derivative securities with the same underlying stock, the prices of the COCB and the ELCB can be calculated by the similar manner. This leads to two coupled PDEs for our pricing model (see Appendix A for a detailed derivation). Let \( U \) be the price of the COCB, and then the price of the ELCB can be obtained as \( V - U \). The coupled PDEs are

\[
\frac{\partial U}{\partial t} + \frac{\partial U}{\partial \tau} d\tau + \frac{1}{2} \sigma^2 S^2 \frac{\partial^2 U}{\partial S^2} + \frac{1}{2} \rho \sigma^2 \frac{\partial^2 U}{\partial r^2} + \rho S \sigma \frac{\partial^2 U}{\partial S \partial r} - (r + r_c)U + rS \frac{\partial U}{\partial S} + K(t) = -(u - \lambda \sigma) \frac{\partial U}{\partial r} \tag{7}
\]

and

\[
\frac{\partial V}{\partial t} + \frac{\partial V}{\partial \tau} d\tau + \frac{1}{2} \sigma^2 S^2 \frac{\partial^2 V}{\partial S^2} + \frac{1}{2} \rho \sigma^2 \frac{\partial^2 V}{\partial r^2} + \rho S \sigma \frac{\partial^2 V}{\partial S \partial r} - r(V - U) - (r + r_c)U + rS \frac{\partial V}{\partial S} + K(t) = -(u - \lambda \sigma) \frac{\partial V}{\partial r} \tag{8}
\]

2.2 The final and boundary conditions

The formulation of our valuation problem is completed by specifying the appropriate final and boundary conditions that are applied at the boundaries of each state space, e.g., at expiration \( t = T \), at the barrier window \( \tau = D \) (\( D \) is the barrier time triggering parameter, i.e., the option is triggered only if \( \tau \geq D \)), and at the lower and upper bounds of variables \( r \) and \( S \).

A general convertible bond is maturing at time \( T \), convertible at any time after \( T_{cv} \) to \( k \) shares of stock, paying a principal \( B \) at maturity if not converted, paying the \( i^{th} \) fixed coupon amount \( K_i \) at times \( t_i \), callable by the issuer at a price \( B_c \) at any time after \( T_c \) if \( \tau \geq D^0 \).

\footnote{Puttable boundary conditions can also be specified similar to callable ones. Without loss of generality, we suppress here by assuming that the bond is only callable.}
Given this setup, similar to Brennan and Schwartz (1980) and Tsiveriotis and Fernandes (1998), the final conditions at maturity are:

\[ V(S, r, T, \tau; T) = \begin{cases} 
  kS & \text{if } kS \leq (B + K_T) \\
  B + K_T & \text{otherwise}
\end{cases} \]  

(9)

\[ U(S, r, T, \tau; T) = \begin{cases} 
  0 & \text{if } kS \geq (B + K_T) \\
  B + K_T & \text{otherwise}
\end{cases} \]  

(10)

The boundary conditions due to conversion are:

\[ V(S, r, t, \tau; T) \geq kS \quad \text{for } t \in [T_v, T] \]  

(11)

\[ U(S, r, t, \tau; T) = 0 \quad \text{if } V \leq kS, \text{ for } t \in [T_v, T]. \]  

(12)

The boundary conditions due to the callability of convertible bonds, which are effective only for \( \tau \geq D \), are the following:

\[ V(S, r, t, \tau; T) \leq \max(kS, B_c) \quad \text{for } t \in [T_c, T] \]  

(13)

\[ U(S, r, t, \tau; T) = 0 \quad \text{if } V \geq B_c, \text{ for } t \in [T_c, T]. \]  

(14)

Further, to insure the continuity of the solution of Parisian options, a pathwise continuity condition for \( S = L \) has to be imposed:

\[ V(L, r, t, \tau; T) = V(L, r, t, 0; T) \]  

(15)

\[ U(L, r, t, \tau; T) = U(L, r, t, 0; T). \]  

(16)

2.3 The interest rate model

The specific form of the interest rate model has to be chosen for valuation. Brennan and Schwartz (1980) and Ammann et al. (2008) find a small effect of the stochastic interest rate on the fair price of a convertible bond. Nevertheless, to complete our two factor model, we choose a general affine model, i.e., the Cox-Ingersoll-Ross (CIR) model (Cox et al. 1985).

The dynamics of the interest rate follow a square root process:
\[ u - \lambda \sigma = a - br, \quad (17) \]
\[ \sigma = \sigma_r r^{1/2}, \quad (18) \]

where \( a, b, \) and \( c \) are constants and (17) is the risk adjusted drift. Plugging (17) and (18) into our pricing equations (7) and (8), we get the following two groups of coupled PDEs depending on the stock price. When the stock price is below the barrier, i.e., \( S < L \) and \( dt = 0 \),
\[
\frac{\partial V}{\partial t} + \frac{1}{2} \sigma^2 S^2 \frac{\partial^2 V}{\partial S^2} + \frac{1}{2} \sigma^2 r \frac{\partial^2 V}{\partial r^2} + \rho S \sigma \sigma_r \sqrt{r} \frac{\partial^2 V}{\partial S \partial r} - r(V - U) - (r + r_c)U + r S \frac{\partial V}{\partial S} + K(t) = -(a - br) \frac{\partial V}{\partial r} \quad (19)
\]
\[
\frac{\partial U}{\partial t} + \frac{1}{2} \sigma^2 S^2 \frac{\partial^2 U}{\partial S^2} + \frac{1}{2} \sigma^2 r \frac{\partial^2 U}{\partial r^2} + \rho S \sigma \sigma_r \sqrt{r} \frac{\partial^2 U}{\partial S \partial r} .
\]
\[
(r + r_c)U + r S \frac{\partial U}{\partial S} + K(t) = -(a - br) \frac{\partial U}{\partial r} \quad (20)
\]

When the stock price is above the barrier, i.e., \( S > L \) and \( dt = dt \),
\[
\frac{\partial V}{\partial t} + \frac{\partial V}{\partial \tau} + \frac{1}{2} \sigma^2 S^2 \frac{\partial^2 V}{\partial S^2} + \frac{1}{2} \sigma^2 r \frac{\partial^2 V}{\partial r^2} + \rho S \sigma \sigma_r \sqrt{r} \frac{\partial^2 V}{\partial S \partial r} - r(V - U) - (r + r_c)U + r S \frac{\partial V}{\partial S} + K(t) = -(a - br) \frac{\partial V}{\partial r} \quad (21)
\]
\[
\frac{\partial U}{\partial t} + \frac{\partial U}{\partial \tau} + \frac{1}{2} \sigma^2 S^2 \frac{\partial^2 U}{\partial S^2} + \frac{1}{2} \sigma^2 r \frac{\partial^2 U}{\partial r^2} + \rho S \sigma \sigma_r \sqrt{r} \frac{\partial^2 U}{\partial S \partial r} .
\]
\[
(r + r_c)U + r S \frac{\partial U}{\partial S} + K(t) = -(a - br) \frac{\partial U}{\partial r} \quad (22)
\]

For both cases, we need to solve the pricing equations simultaneously because they are inter-correlated. This relationship stems from the fact that the convertible bond valuation that we are facing is a free boundary problem and the boundary conditions for the ELCB define the boundary conditions for the COCB.

### 2.4 Numerical solution by a finite element method

A convertible bond can now be valued by solving partial differential equations (19) to (22) subject to conditions (9) to (16). Two classes of methods for this problem include finite
difference methods and finite element methods. We apply a finite element method as it gives convergent deterministic approximations under the realistic and low smoothness assumptions on the payoff function. In particular, it allows a higher rate of convergence. A standard transformation is taken for equations (19) to (22) by letting \( x = \ln S, \hat{t} = T - t, \hat{\tau} = D - \tau, \) such that \( S \) is eliminated in the coefficients and the diffusion equations are simplified:

For \( S < L, \)

\[
\frac{\partial V}{\partial \hat{t}} - \frac{1}{2} \sigma^2 \frac{\partial^2 V}{\partial x^2} - \frac{1}{2} \sigma^2 r \frac{\partial^2 V}{\partial r^2} - \rho \sigma \sigma, \sqrt{r} \frac{\partial^2 V}{\partial x \partial r} + r(V - U) + (r + r_c)U - \left( r - \frac{1}{2} \sigma^2 \right) \frac{\partial V}{\partial x} - K(\hat{t}) = (a - br) \frac{\partial V}{\partial r}.
\]

(23)

For \( S > L, \)

\[
\frac{\partial V}{\partial \hat{t}} + \frac{\partial V}{\partial \hat{\tau}} - \frac{1}{2} \sigma^2 \frac{\partial^2 V}{\partial x^2} - \frac{1}{2} \sigma^2 r \frac{\partial^2 V}{\partial r^2} - \rho \sigma \sigma, \sqrt{r} \frac{\partial^2 V}{\partial x \partial r} + r(V - U) + (r + r_c)U - \left( r - \frac{1}{2} \sigma^2 \right) \frac{\partial V}{\partial x} - K(\hat{t}) = (a - br) \frac{\partial V}{\partial r}.
\]

(24)

\[
\frac{\partial U}{\partial \hat{t}} + \frac{\partial U}{\partial \hat{\tau}} - \frac{1}{2} \sigma^2 \frac{\partial^2 U}{\partial x^2} - \frac{1}{2} \sigma^2 r \frac{\partial^2 U}{\partial r^2} - \rho \sigma \sigma, \sqrt{r} \frac{\partial^2 U}{\partial x \partial r} + (r + r_c)U - \left( r - \frac{1}{2} \sigma^2 \right) \frac{\partial U}{\partial x} - K(\hat{t}) = (a - br) \frac{\partial U}{\partial r}.
\]

(25)

\[
\frac{\partial U}{\partial \hat{t}} + \frac{\partial U}{\partial \hat{\tau}} - \frac{1}{2} \sigma^2 \frac{\partial^2 U}{\partial x^2} - \frac{1}{2} \sigma^2 r \frac{\partial^2 U}{\partial r^2} - \rho \sigma \sigma, \sqrt{r} \frac{\partial^2 U}{\partial x \partial r} + (r + r_c)U - \left( r - \frac{1}{2} \sigma^2 \right) \frac{\partial U}{\partial x} - K(\hat{t}) = (a - br) \frac{\partial U}{\partial r}.
\]

(26)

The numerical solution proceeds as follows. First, equations (23) and (24) are used to calculate the value of \( U \) and \( V \) when \( S < L \). Second, the continuity conditions (15) and (16) are imposed for the case when \( S = L \). Finally, equations (25) and (26) are applied when \( S > L \).

It should be noted that (23) and (25) are computed prior to (24) and (26) for the discretization convenience.

Barone-Adesi et al. (2003) propose the method of characteristics together with finite elements for valuing a two-factor convertible bond. Following them, we solve the PDEs with
a finite element method by finding variational formulation and introducing a $\theta$-scheme for the pricing equations (see Appendix B for a detailed description).

3. **The data**

3.1. **Convertible bonds**

We choose to investigate the Chinese domestic market (including Shanghai and Shenzhen Stock Exchange) from Mar 2, 2006, to Feb 1, 2010. Our convertible bond sample consists of 49 bonds with a total of 2469 observations at the weekly frequency. Table 1 gives an overview of the analyzed convertible bonds.

The average market capitalization is about 1.18 billion RMB (or $170 million). Typically, the coupon rate is not constant across years. The average coupon rate is 1.86% and there is little variation across bonds. Time to maturity is calculated as the time between the bond’s first observation in our sample and maturity. The average time to maturity is 46 months, close to the duration of the sample. It means that almost half of the bonds in the sample are mature before our sampling period ends. The call and put options are embedded for most of the bonds in China. Particularly, 48 out of 49 analyzed bonds include a call option, allowing the issuer to repurchase the bond for a certain price $B_c$, called “call price” or “early redemption price”. All the 49 analyzed bonds include a put option, allowing the bond holder to sell the bond for a certain price $B_p$, called “put price”. Both of the call and put options have the Parisian feature that requires the stock price must stay above the upper barrier (for the call option) or below the lower barrier (for the put option) for a consecutive time before the option is activated. The prevailing presence of the Parisian feature in the Chinese market is also one of the reasons that we choose the Chinese dataset. In contrast, the US data used in Ammann et al. (2008) include only 2 convertibles with the Parisian feature out of total 32 convertibles. Presumably, the impact of the Parisian feature on pricing convertible bonds is
stronger for the Chinese data than the US data. The consecutive time, called “barrier window”, is on average 22 days for the call option and 25 days for the put option. This number is somewhat overestimated because the trigger conditions of some bonds only require that the bond price stays above the upper barrier for 20 days in consecutive 30 days. In these cases, the barrier window is approximately set to 20 days. Such simplification is also assumed when we empirically price the bonds. The upper barrier and lower barrier are on average 131% and 73% of the conversion price (the ratio of face value and the conversion ratio), respectively, with very little difference across bonds. The call price and put price are 103% and 104% of the face value. These characteristics are extracted for every bond and are used as the input parameters for pricing the bonds one by one.

3.2. Stock

The input parameters related to the stock data are stock prices and volatilities. Although stock prices are directly observable, the volatility of the underlying stock needs to be estimated. The methods on estimating the stock volatility have a rich literature. A popular approach is the implied volatility concept. However, stock options are absent in the Chinese market. We therefore estimate volatility on a historical basis. Like other emerging markets, Chinese stocks typically have high volatilities. In order to track changes in the volatility, we use the exponentially weighted moving average (EWMA) model by RiskMetrics with \( \lambda = 0.94 \).\(^{10}\) The formula is

\[
\sigma_n^2 = \lambda \sigma_{n-1}^2 + (1 - \lambda) u_{n-1}^2.
\]  

(27)

The estimate, \( \sigma_n \), of the volatility for day \( n \) is calculated from \( \sigma_{n-1} \) and \( u_{n-1} \) (the most recent daily percentage change). We calculate the volatility based on the last 250 trading days and update the input volatility for every calculation of the theoretical bond price. Figure 1

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\(^{10}\) See J. P. Morgan, 1995, RiskMetrics Monitor, Fourth Quarter.
presents the distribution of the average of all annualized input volatilities that are used in our empirical analysis. The mean volatility of all the input volatilities is 51%. This estimate is coincidently the same as the average volatility across the US data in Ammann et al. (2008).

3.3. Interest rate and credit spreads

The CIR interest rate model in (2), (17), and (18) is calibrated to Chinese treasury bond yield curves from Mar 1, 2006 to Feb 1, 2010. Figure 2 presents the estimated parameters of the CIR model for every trading day in the sample. On average, the estimate of $\sigma_r$ is 0.029, $\alpha$ is 0.169, and $\beta$ is 0.043.

Since most of the issuers do not have straight debt in the market, the credit spread is calculated as the difference between yield curves of risk free treasury bonds and corporate bonds, e.g. the Chinabond Treasury Yield Curve and the Chinabond Corporate Yield Curve.

The last input parameter is $\rho$, the correlation between the stock process (1) and the interest rate process (2). We use the correlation between the stock market index and bond market index as a proxy for this parameter.

4. Results

We numerically solve the theoretical price for each bond in our sample on a weekly frequency. The observed convertible bond prices in the Chinese market are then compared with the theoretical prices. The percentage overpricing is calculated as the difference between the market and theoretical price over the market price. The mean percentage overpricing is presented for each convertible bond in Table 2. In order to quantify the empirical importance of the Parisian feature and stochastic interest rate, we also calculate the mean percentage overpricing for three alternative models. Therefore, we have (a) the benchmark model with the Parisian option and stochastic interest rate; (b) the model with the Parisian option and
with constant interest rate; (c) the model without the Parisian option and with stochastic interest rate; (d) the model without the Parisian option and with constant interest rate. The constant interest rate is linearly extrapolated from the Chinabond treasury yield curve. For each bond, a two-sided test is conducted for the $H_0$ hypothesis that theoretical prices of the benchmark model and market prices are equal in the mean. The last column presents the standard deviation of the mean squared percentage pricing deviation between the benchmark model and market.

The mean percentage overpricing for a single bond ranges from -7.84% to 11.53%. Our benchmark model detects 31 cases of underpricing and 18 cases of overpricing. The null hypothesis that the difference of the model and market prices is equal to zero is rejected at a ten percent significance level for 26 cases of underpricing and 17 cases of overpricing. It seems underpricing dominates the Chinese convertible bond market. When we calculate the average overpricing across the bonds, however, the overpricing is -0.001%. The market does not show an underpricing on average as documented in the literature. Our result is in line with the recent study of Ammann et al. (2008) which does not confirm the evidence of previous studies that market prices of convertible bonds are on average lower than prices generated by a theoretical model.

The corresponding percentage overpricing for the alternative models (b), (c), and (d) are 0.23%, 5.61%, and 5.75%, respectively. Ignoring the Parisian options can increase the overpricing from almost zero to 5.61% with stochastic interest rate or from 0.23% to 5.75% with constant interest rate. The pricing effect is more than 5%. The large pricing effect of the Parisian option is striking because the overpricing in the literature ranged from -12.9% to 1.7%. None of the empirical studies in the literature have considered the Parisian option explicitly. If the Parisian option were included in their studies, the underpricing would be 5% more. However, we ought to be cautious in extending the result of the Chinese market to
other markets for two reasons. First, most convertible bonds in some markets like US do not have the Parisian options embedded. Hence, the effect of the Parisian option is irrelevant in these markets. Second, the effect is increasing in the volatility of the market. In markets with low volatilities, the pricing effect is expected to be small. Nevertheless, the result of the present paper suggests that ignoring the Parisian option embedded in convertible bonds will significantly underestimate the theoretical prices in emerging markets with high volatility. On the other hand, the marginal contribution of the stochastic interest rate to the overpricing is 0.23% with the Parisian option and 0.14% without the Parisian option. This rather small effect confirms the findings in Brennan and Schwartz (1980) and Ammann et al. (2008) that stochastic interest rate is of second order importance when empirically pricing convertible bonds.

Some bonds have less than one year in our sample. The estimate of the mean percentage overpricing may suffer from the small sample. Therefore we select the bonds with longer than one year to form a subsample of 21 bonds. The mean percentage overpricing for this subsample is 0.99%, i.e. the market prices are still slightly above the theoretical prices of the benchmark model. Though mispricing is present after we delete the bonds with short periods, the evidence is yet against the long-standing underpricing puzzle in the literature. The overpricing is even stronger than the full sample.

5. Conclusion

We conduct a pricing study for the Chinese convertible bond market in this paper. The Parisian options are popularly embedded in convertible bonds in China and other markets. The previous studies, however, have not considered the Parisian option when calculating the theoretical prices. This is the first study modeling the Parisian option and stochastic interest rate together and applying the model to a market in which the Parisian option prevails. We
propose a two-factor model and study a period of 47 months using weekly data. We find that theoretical values for the analyzed convertible bonds will be underestimated by more than 5% if we ignore the Parisian options. More than half of the analyzed bonds exhibit a statistically significant mean underpricing. The average underpricing across bonds, however, does not indicate a mispricing in the Chinese market. Similar to the recent study by Ammann et al. (2008), our results do not confirm the evidence of previous studies that market prices of convertible bonds are on average lower than prices generated by a theoretical model.
Appendix A: Derivation of the pricing PDEs

This appendix derives the coupled pricing PDEs (7) and (8).

Given the bond price $V = V(S, r, t, \tau; T)$ and the stochastic processes (1) and (2), the stochastic process of $V$ can be derived using Ito’s lemma:

$$
\begin{align*}
    dV &= \frac{\partial V}{\partial t} dt + \frac{\partial V}{\partial \tau} d\tau + \frac{\partial V}{\partial S} dS + \frac{1}{2} \frac{\partial^2 V}{\partial S^2} (dS)^2 + \frac{\partial V}{\partial r} dr + \frac{1}{2} \frac{\partial^2 V}{\partial r^2} (dr)^2 + \rho \frac{\partial^2 V}{\partial S \partial r} (dS dr) \\
    &= \left[ \frac{\partial V}{\partial t} + \frac{\partial V}{\partial \tau} \frac{d\tau}{dt} + \frac{1}{2} \sigma^2 S^2 \frac{\partial^2 V}{\partial S^2} + \frac{1}{2} \sigma^2 \frac{\partial^2 V}{\partial r^2} + \rho S \sigma \frac{\partial^2 V}{\partial S \partial r} \right] dt + \frac{\partial V}{\partial S} dS + \frac{\partial V}{\partial r} dr.
\end{align*}
\tag{A1}
$$

Let us construct a portfolio $\Pi$ consisting of one convertible bond $V, -\Delta_1$ shares of another convertible bond $V_1$ and $-\Delta$ shares of the underlying stock $S$. Therefore, we get the equation (5)

$$
    d\Pi = dV - \Delta_1 dV_1 - \Delta dS.
$$

To eliminate the risk source of this portfolio, we must choose

$$
    \Delta_1 = \frac{\partial V}{\partial V_1} / \frac{\partial V}{\partial S}, \quad \Delta = \frac{\partial V}{\partial V_1} / \frac{\partial V}{\partial S}.
$$

The change of the portfolio value is therefore given by

$$
\begin{align*}
    d\Pi &= \left[ \frac{\partial V}{\partial t} + \frac{\partial V}{\partial \tau} \frac{d\tau}{dt} + \frac{1}{2} \sigma^2 S^2 \frac{\partial^2 V}{\partial S^2} + \frac{1}{2} \sigma^2 \frac{\partial^2 V}{\partial r^2} + \rho S \sigma \frac{\partial^2 V}{\partial S \partial r} \right] dt \\
    &\quad - \Delta_1 \left[ \frac{\partial V_1}{\partial t} + \frac{\partial V_1}{\partial \tau} \frac{d\tau}{dt} + \frac{1}{2} \sigma^2 S^2 \frac{\partial^2 V_1}{\partial S^2} + \frac{1}{2} \sigma^2 \frac{\partial^2 V_1}{\partial r^2} + \rho S \sigma \frac{\partial^2 V_1}{\partial S \partial r} \right] dt.
\end{align*}
\tag{A2}
$$

Since this portfolio is under the no-arbitrage condition, it should generate a risk-free return

$$
    d\Pi = r \Pi dt = r(V - \Delta V_1 - \Delta S) dt
$$

Collecting all terms of $V$ on the left side and all terms of $V_1$ on the right side yields

---

11 See Wilmott et al. (1993).
\[ \frac{\partial V}{\partial t} + \frac{\partial V}{\partial \tau} \frac{d\tau}{dt} + \frac{1}{2} \sigma^2 S^2 \frac{\partial^2 V}{\partial S^2} + \frac{1}{2} \sigma^2 \frac{\partial^2 V}{\partial r^2} + \rho \sigma \rho S \sigma \rho \frac{\partial^2 V}{\partial S \partial r} - r V + r S \frac{\partial V}{\partial S} = \frac{\partial V}{\partial r} \cdot \tag{A3} \]

Since the left and right hand sides depend only on \( V \) and \( V_1 \) respectively, the only possible solution is that both sides are equal to a function of independent variables. Analogous to Heston (1993), we choose a function

\[ \frac{\partial V}{\partial t} + \frac{\partial V}{\partial \tau} \frac{d\tau}{dt} + \frac{1}{2} \sigma^2 S^2 \frac{\partial^2 V}{\partial S^2} + \frac{1}{2} \sigma^2 \frac{\partial^2 V}{\partial r^2} + \rho \sigma \rho S \sigma \rho \frac{\partial^2 V}{\partial S \partial r} - r V + r S \frac{\partial V}{\partial S} = -(u - \lambda \omega) \frac{\partial V}{\partial r} \cdot \tag{A4} \]

where \( \lambda \) is the market price of interest rate risk.

More generally, when the bond pays coupons \( K(t) \), we have

\[ \frac{\partial V}{\partial t} + \frac{\partial V}{\partial \tau} \frac{d\tau}{dt} + \frac{1}{2} \sigma^2 S^2 \frac{\partial^2 V}{\partial S^2} + \frac{1}{2} \sigma^2 \frac{\partial^2 V}{\partial r^2} + \rho \sigma \rho S \sigma \rho \frac{\partial^2 V}{\partial S \partial r} - r V + r S \frac{\partial V}{\partial S} + K(t) = -(u - \lambda \omega) \frac{\partial V}{\partial r} \cdot \tag{A4} \]

This completes equation (6).

We would expect that besides the dynamics in (1), the value of a risky bond COCB, loses a fraction per time step. The loss is measured by \( r_c dt \), where \( r_c \) is the credit spread.

Therefore we adjust the equation (A1) to obtain the following equation for a COCB

\[ \frac{dU}{dt} = \left[ \frac{\partial U}{\partial t} + \frac{\partial U}{\partial \tau} \frac{d\tau}{dt} + \frac{1}{2} \sigma^2 S^2 \frac{\partial^2 U}{\partial S^2} + \frac{1}{2} \sigma^2 \frac{\partial^2 U}{\partial r^2} + \rho \sigma \rho S \sigma \rho \frac{\partial^2 U}{\partial S \partial r} \right] dt + \frac{\partial U}{\partial S} dS + \frac{\partial U}{\partial r} dr - r_c U dt \cdot \tag{A5} \]

Applying the same no-arbitrage argument above, we end up with the pricing equation (7) for a COCB

\[ \frac{\partial U}{\partial t} + \frac{\partial U}{\partial \tau} \frac{d\tau}{dt} + \frac{1}{2} \sigma^2 S^2 \frac{\partial^2 U}{\partial S^2} + \frac{1}{2} \sigma^2 \frac{\partial^2 U}{\partial r^2} + \rho \sigma \rho S \sigma \rho \frac{\partial^2 U}{\partial S \partial r} - (r + r_c) U + r S \frac{\partial U}{\partial S} + K(t) = -(u - \lambda \omega) \frac{\partial U}{\partial r} \cdot \]
Similarly, we can obtain equation (8) for pricing an ELCB, which is just the difference between $V$ and $U$.

\[
\frac{\partial V}{\partial t} + \frac{\partial V}{\partial \tau} \frac{d \tau}{dt} + \frac{1}{2} \sigma^2 S^2 \frac{\partial^2 V}{\partial S^2} + \frac{1}{2} \sigma^2 \frac{\partial^2 V}{\partial r^2} + \rho S \frac{\partial^2 V}{\partial S \partial r} - r(V - U) - (r + r_j)U + rS \frac{\partial V}{\partial S} + K(t) = -(u - \lambda) \frac{\partial V}{\partial r} .
\]

**Appendix B: Numerical procedure**

This appendix describes the numerical procedure to solve the PDEs (23) to (26) by applying a finite element method following Barone-Adesi et al. (2003). We use equation (25) as an example to illustrate. (25) can be re-written as\(^{12}\)

\[
\frac{\partial}{\partial t} \hat{V} + \frac{\partial}{\partial \tau} \hat{V} + A'' V = 0 ,
\]

with operators given by

\[
A'' = -\frac{1}{2} \sigma^2 \hat{\sigma} - \frac{1}{2} \sigma^2 r \hat{\sigma}_r - \rho \sigma \hat{\sigma}_r \sqrt{r} \hat{\sigma}_{rr} + r(V - U) + (r + r_j)U - (r - \frac{1}{2} \sigma^2 \hat{\sigma}) \hat{\sigma}_x - K(t) - (a - br) \hat{\sigma}_r .
\]

The variational formulation is derived by finding $V \in L^2(0,T) \cap H^1(0,T)$ and a test function $z$ such that

\[
\left( \frac{d}{dt} \hat{V}(\hat{t}), z \right) + \left( \frac{d}{d\hat{t}} V(\hat{\tau}), z \right) + \alpha''(V, z) = 0 ,
\]

where

---

\(^{12}\) $\partial_x y$ denotes the first order derivative of $y$ with respect to $x$ and $\partial_{xx} y$ denotes the second order derivative of $y$ with respect to $x$. 
\[ \alpha''(V,z) = -\frac{1}{2}\sigma^2 \int_{\Omega} \partial_{xx} V z dxdr - \frac{1}{2}\sigma^2 \int_{\Omega} r \partial_{x} V z dxdr - \rho \sigma \sigma_r \int_{\Omega} \sqrt{r} \partial_{x} V z dxdr \\
- \int_{\Omega} (r - \frac{1}{2}\sigma^2) \partial_{x} V z dxdr - \int_{\Omega} (a - br) \partial_{x} V z dxdr + \int_{\Omega} r(V - U) z dxdr + \int_{\Omega} (r + r_c) V z dxdr - K(i) \int_{\Omega} z dxdr \]

using integration by parts.

The time dimension is discretized by the 0-Scheme with time step \( k = \frac{T}{M_1} = \frac{D}{M_2} \) where \( M_1 \) and \( M_2 \) are the numbers of time steps for the maturity and for the barrier window\(^{13}\). The space dimension by the finite element space\(^{14}\)

\[ I_{N_x,N_y} = \text{span} \{ b_i(x)b_j(r) \mid i = 1,\ldots,N_x; j = 0,\ldots,N_r + 1 \} \]

of continuous, piecewise bilinears with respect to the two-dimensional grid \((x_i, r_l)\),

\[ x_i = x_{\min} + i h_x, r_l = l h_r, \text{ with mesh width } h_x = \frac{x_{\max} - x_{\min}}{N_x + 1}, h_r = \frac{r_{\max}}{N_r + 1}, \text{ and } \]

\[ b_i(x) = \max(1 - \frac{|x - x_i|}{h_x}, 0), b_j(r) = \max(1 - \frac{|r - r_l|}{h_r}, 0). \] Noting here by choosing the 0-Scheme, Crank-Nicolson method can be easily incorporated by setting \( \theta = 0.5 \), which is widely accepted more accurate than the explicit or implicit finite difference alone. Denoting \( V_{i,j}^n \) the numerical approximation of the convertible bond value at share price \( S = i \Delta S \), time to expiration \( n \Delta t \) and time to barrier window \( j \Delta \tau \), we have

\[ \frac{\partial V_i(t,\tau)}{\partial t} \approx \frac{V_{i,j}^{n+1} - V_{i,j}^n}{k}, \quad \frac{\partial V_i(t,\tau)}{\partial \tau} \approx \frac{V_{i,j+1}^{n+1} - V_{i,j}^{n+1}}{k}. \] Equation (25) can then be expressed as the matrix formulation

\(^{13}\) For simplicity, the time interval for \( T \) and \( D \) are set to be the same.

\(^{14}\) \( N \) is the number of steps for discretization.
\[
\frac{1}{k} M (V_{n+1}^j - V_j^n) + \theta A V_{n+1}^j + (1 - \theta) A V_j^n = 0
\]

\[
(M + k \theta A) V_j^{n+1} = (M - k (1 - \theta) A) V_j^n
\]  

(A8)

where \( V_j^n = (V_{1,j}^n, V_{2,j}^n, \ldots, V_{i,j}^n) \) is a vector of convertible bond prices. Solving (A8) yields the convertible bond fair prices for our empirical study.
References


Table 1: Overview of the convertible bonds

This table gives an overview of the analyzed convertible bonds with size, maturity, coupon, and Parisian features of call and put options. Columns 2 to 6 list the percentiles for the corresponding continuous variables.

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<th></th>
<th>Mean</th>
<th>5%</th>
<th>25%</th>
<th>50%</th>
<th>75%</th>
<th>95%</th>
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<tbody>
<tr>
<td>Market capitalization (billion RMB)</td>
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<td>0.80</td>
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<td>103</td>
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</table>

*Since most of the convertible bonds have an ascending coupon rates across years, we calculate the average coupon rate.

**Time to maturity is calculated as the time between the bond’s first observation in our sample and maturity.

***The trigger conditions of some bonds only require that the bond price stays above the upper barrier for 20 days in consecutive 30 days. In these cases, the barrier window is approximately set to 20.
Table 2: Mispricing of the convertible bonds

This table gives an overview of the percentage overpricing of the convertible bonds. Columns (a) to (d) present the mean percentage overpricing for each bond. Specifically, we have Model (a) the Parisian option and stochastic interest rate; Model (b) with the Parisian option and with constant interest rate; Model (c) without the Parisian option and with stochastic interest rate; Model (d) without the Parisian option and with constant interest rate. For each bond, a two-sided test is conducted for the $H_0$ hypothesis that the theoretical prices of Model (a) and market prices are equal in the mean. ***Significant at 1%; **significant at 5%; *significant at 10%. The root mean squared error is the standard deviation of the mean squared percentage pricing error of Model (a) from market prices.

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<th>(b) %</th>
<th>(c) %</th>
<th>(d) %</th>
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Figure 1: Distribution of input volatilities

This figure presents the distribution of the average of all annualized input volatilities that are used in the empirical analysis.
Figure 2: Input parameters of the CIR model

This figure presents the estimated parameters of the CIR model for every trading day in the sample.